DOCUMENT RESUME

ED 081 375 HE 004 558

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TITLE The Production and Cost Behavior of Higher Education Institutions.
INSTITUTION California Univ., Berkeley. Ford Foundation Program for Research in Univ. Administration.
SPONS AGENCY Ford Foundation, New York, N.Y.
REPORT NO Pap-P-36
PUB DATE Dec 72
NOTE 191p.
AVAILABLE FROM Ford Foundation, 2288 Fulton Street, Berkeley, California 94720

EDRS PRICE MF-$0.65 HC-$6.58
DESCRIPTORS *Costs; Doctoral Theses; *Educational Finance; *Higher Education; *Productivity; *Student Enrollment

ABSTRACT

This report is an empirical analysis of the "frontier" production and cost relationships between the number of students enrolled and the labor and capital inputs observed over a wide cross-section of four-year higher education institutions in the United States. In the analysis, students are differentiated as to type and as to part-time versus full-time. It is assumed that the production and cost relationships are dependent on other measurable characteristics of the institutions; therefore, several additional variables, such as institutional quality and program mix, are included in the model. Also, separate analyses are performed for each type of institution (public and private universities, public and private comprehensive colleges, and private liberal arts colleges). The empirical results generated by this study indicate that the frontier average and marginal relationships between the institutional input and enrollment variables are complex functions of input structures. enrollment mixes, and several institutional characteristics. This type of economic behavior is discouraging for national policy analysis, since it implies that no simple set of production relationships applies to all institutions. In addition, the results illustrate that the frontier production and cost relationships are not neutral, linear transformations of the average production behavior. An extensive bibliography is included. (Author)

FORD FOUNDATION PROGRAM FOR RESEARCH IN UNIVERSITY ADMINISTRATION

Office of the Vice President—Planning
University of California

FORD GRANT NO. 680-0267A

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THE PRODUCTION AND COST BEHAVIOR OF HIGHER EDUCATION INSTITUTIONS

Daryl E. Carlson

Paper P-36

December 1972

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PREFACE

This is one of a continuing series of reports of the Ford Foundation sponsored Research Program in University Administration at the University of California, Berkeley. The guiding purpose of this Program is to undertake quantitative research which will assist university administrators and other individuals seriously concerned with the management of university systems both to understand the basic functions of their complex systems and to utilize effectively the tools of modern management in the allocation of educational resources.

This report consists of the entire doctoral dissertation (under the same title as this report) submitted to the Graduate School of Business Administration at the University of California, Berkeley, on November 20, 1972. The dissertation is an empirical analysis of the "frontier" production and cost relationships between the number of students enrolled and the labor and capital inputs observed over a wide cross-section of four-year higher education institutions in the United States. The empirical results generated by this study indicate that the frontier average and marginal relationships between the institutional input and enrollment variables are complex functions of input structures, enrollment mixes, and several institutional characteristics. The methodology used in the study uncovers some interesting patterns of *frontier* production and cost behavior for higher education institutions that have not been obtained in econometric studies of *average* production and cost behavior.

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I. INTRODUCTION

The financing of higher education in the United States has become a major problem to be confronted in the 1970's.[1] Total expenditures for higher education have risen sharply during the past decade and are projected to increase even more rapidly over the current decade as shown in Table 1-1. The demand for higher education as well as the diversity of programs and services provided by colleges and universities have also increased markedly over the past 10 years. In the past, colleges and universities met the problem of financing additional and expanded programs by simply seeking more revenue. The overall scale of higher education was small; there were always places to obtain additional funds, and words such as productivity, efficiency, and accountability were seldom mentioned. Through the 1960's, however, higher education has grown much faster than the rest of the economy, as shown in Table 1-1, and schools are now finding it difficult to obtain funds to continue this rapid increase in programs. In fact, there are many cases where the maintenance of existing programs are endangered by the lack of funds.[2]

It is this combination of increased demand for the outputs of higher education along with limited funds that has forced decision makers at colleges, universities, and all levels of government to be concerned about the use of resources in higher education. Education must compete with other high priority national needs (welfare, health care, defense, conservation, . . .) for federal and state monies.

Many cost studies have been completed in the past few years and an extensive bibliography has been compiled by D. Witmer [1972].

TABLE 1-1

The Growth of Higher Education

	1950	1960	1968	1980
Enrollment in Institutions of Higher Education (thousands)	2,286	3,583	6,928	13,277
Expenditures by Institutions of Higher Education (millions)	$2,260	$5,628	$16,566	$43,800
Percent Expenditures are of Gross National Product	0.8%	1.1%	1.9%	3.7%

SOURCES: Digest of Educational Statistics, National Center for Educational Statistics, 1970.

Statistical Abstract of the United States, U.S. Department of Commerce, Bureau of the Census, 1971.

There is also a large bibliography on operations analysis in higher education compiled by R. Schroeder [1972]. To illustrate the need for an analysis of the diverse cost behavior of institutions of higher education, Table 1-2 shows the extreme variation that exists in general educational expenditures per student even within relatively homogeneous groups of institutions. The range of general educational expenditures per student (on a total enrollment basis) is quite wide as exhibited by the minimum and maximum values for each institutional category. Similarly, the coefficient of variation indicates substantial variation within each group of institutions. As illustrated in a later chapter, these variations in costs per student are the result of different enrollment compositions, different institutional characteristics, and inefficiency.

General Framework of the Study

This research is designed with the institution as the basic unit of analysis. The general approach is that of a micro-economic analysis of the production and cost behavior of a particular industry. This study deals with the "higher education" industry and the colleges and universities are treated as the "firms" in the industry. Several researchers and writers on the economics of higher education have made this analogy or taken this approach.[3] The basic assumption is that the higher education "firms" transform some combination and amount of inputs into outputs of some nature.

Using the institution as the unit of analysis is in contrast to other education "production function" studies that have considered departments within an institution or the students themselves as the

TABLE 1-2

Statistics for General Educational Expenditures* per Student** by Institutional Category

Institutional Category	Mean	Standard Deviation	Coefficient of Variation	Minimum	Maximum
Public Universities	2,499	1,070	0.43	887	4,452
Private Universities	4,609	2,911	0.63	812	10,977
Public Comprehensive Colleges	1,144	311	0.27	504	2,508
Private Comprehensive Colleges	1,434	563	0.39	674	3,505
Public Limited Comprehensive Colleges	1,172	277	0.24	643	2,040
Private Highly Selective Liberal Arts Colleges	2,393	831	0.35	988	5,460
Private Less Selective Liberal Arts Colleges	1,494	498	0.33	556	5,980

SOURCE: Office of Education, HEGIS institutional data, fiscal year 1968.

*General Educational Expenditures include the cost of instruction and departmental research, extension and public service, libraries, physical plant maintenance and operation, general administration, general institutional expense and student services, organized activities relating to educational departments, organized research, and other sponsored programs.

**On a total enrollment basis.

basic unit to be studied. Several reasons for this choice of the institution as the basic element are discussed below with references to studies that have used different approaches to studying the economic behavior of institutions of higher education.

First, this study is a financial and economic analysis rather than a sociological-psychological analysis as illustrated in the research by S. Bowles [1970] and H. Levin [1971]. The framework of their studies is constructed around individual students and their differing traits whereas in this study the analysis is structured around the institutions and their varying characteristics. The purpose of the present research is to gain an understanding of the economic behavior of the institutions and therefore the study is concerned with the interrelationships of the different characteristics of the colleges and universities.

Second, this study utilizes cross-sectional data rather than aggregate time-series data as used in the recent study of resource use in higher education by J. O'Neill [1971]. Since it is the diversity in the behavior of institutions that is studied, cross-sectional data provide more information for this purpose. Aggregate time-series data conceal many of the relationships that are being studied. Also, time-series data on a single institution do not exhibit as wide a range of behavior as data for one year from a large cross-section of colleges and universities. Since one aspect of the study is to make inferences about the behavior of all four-year institutions of higher education in the United States, it is necessary to include a large number of these institutions into the empirical analysis. It would be very unreliable to draw conclusions about all colleges and universities from the analysis of only one or two selected institutions.

Third, this study is based on an estimation approach rather than an engineering approach. In other words, a model is developed and estimated rather than constructing separate relationships between each input and output variable for each institution. This latter approach is used in activity analysis or other schemes of analysis that are based on the direct allocation of inputs to each output produced. Unit-cost studies are also based on this type of approach since costs are broken down as fine as possible and then allocated to each specific output. The allocated costs are totaled for each output in order to compute the cost of a single unit of the specified product. The work being done at the National Center for Higher Education Management Systems at the Western Interstate Commission for Higher Education (NCHEMS at WICHE) is typical of this approach to unit-cost analysis. Activity analysis of faculty time has also been the subject of many studies and involves the allocation of faculty time to each activity of the institution (undergraduate instruction, graduate instruction, research, and public service).[4]

Fourth, this study is not a cost-benefit analysis. Primary emphasis is on the expenditures by institutions, and such costs as the students' time or other social costs are not included. The costs that are excluded from the study are not considered unimportant; they are only beyond the scope of this research. Again the emphasis is on the economic behavior of the institutions and not an evaluation of higher education with respect to society in general. No attempt is made to place dollar values on the benefits of higher education since comparisons between "costs" and "benefits" are not performed. The article by M. Woodhall [1970] illustrates an approach used and the problems encountered in a cost-benefit analysis of higher education.

Outline of the Study

Various aspects of the economic behavior of higher education institutions are discussed in Chapter II, providing the economic framework for the empirical analysis performed in the study. In Chapter III the sources of data and the definitions of variables are presented along with a description of the sample of institutions and illustrations of the diversity of the data across and within the institutional categories. The computational method is developed in the fourth chapter and compared to other estimation techniques. These first four chapters complete the foundation for the empirical research done in the study.

The remaining chapters describe the empirical results generated from the model, the data, and the estimation method described in the first four chapters. In Chapter V, descriptive results of the frontier production and cost behavior of higher education institutions are presented. The sensitivity of these calculated relationships is discussed and illustrated in Chapter VI. In the seventh chapter, several hypotheses about the behavior of costs and production variables are formulated and examined. This analysis also illustrates the way in which the empirical approach developed in this study can be used to answer some interesting questions about higher education. Some of the common production and cost relationships are summarized in the final chapter and some general conclusions about cost analysis in higher education are presented.

Some Implications of the Results

The results generated by this empirical analysis of higher education institutions indicate that the "frontier" production and cost relationships between the institutional input and enrollment variables are complex functions of input structures, enrollment mixes, and several institutional characteristics. For example, "frontier" student-faculty ratios are shown to be greater for institutions with either a low quality rating, a small number of programs, a large scale, few graduate students, or few specialized students. Similar results are generated for total costs per student. Although both student-faculty ratios and costs per student are often used as yardsticks in comparing the performance of institutions, the results from this study show that the "frontier" levels of these ratios vary with the levels of institutional inputs, enrollments, and characteristics (institutional quality, percent science degrees, enrollment retention, institutional scale, number of programs, enrollment growth, research commitment, and public service involvement). Therefore, institutional comparisons should be made with all of these factors taken into account.

The results also show that the variations in costs per student due to differences in enrollment mixes and institutional characteristics are as great or greater than the variations due to inefficiency relative to the observed cost frontiers. Enrollment mix and the institutional characteristics are determined primarily by the mission and goals of the institution. The results indicate that what an institution produces is as important as how the institution produces in determining the resulting cost and production behavior. The observed frontier production behavior implies that cost-reducing strategies may not be universally applicable to all institutions; for example, cost reduction may require a different input structure for a high quality institution than for a low quality institution. Also "least-cost per student" institutions are not necessarily the institutions with the largest student-faculty ratios; all of the institutional inputs need to be considered in the determination of least-cost input structures.

The hypothesis that higher education institutions should be analyzed as joint production processes is substantiated by the results of this study. For example, the institutional resources required for an additional full-time undergraduate depends on the institution's current level and mix of enrollment and its current research commitment and public service involvement. The results indicate that it is not valid to isolate one activity of the institution and to analyze the production and cost relationships associated with that activity in isolation from all the other activities of the institution.

A detailed comparison of the production and cost behavior of public versus private institutions leads to the conclusion that the private institutions are being under-utilized. With few exceptions, the marginal product and marginal cost relationships indicate that the private "frontier" institutions could actually decrease their average costs per student by increasing enrollment, whereas the public "frontier" institutions would experience an <u>increase</u> in average costs per student with increased enrollment. Although private institutions are usually shown to have higher per student costs than public institutions, the results from this study show that when enrollment mix and institutional characteristic variables such as quality, program mix, and scale are taken into account, the private "frontier" institutions actually exhibit <u>lower</u> per student costs.

The empirical results also indicate a considerable distance between the <u>average</u> production and cost relationships and the <u>frontier</u> relationships. Upon which relationships should policies be based? For example, should government funds be allocated to institutions on the basis of average or frontier resource requirements? Since the frontier relationships are observed to be non-neutral transformations of the average relationships, implications for the appropriate structure of inputs (in addition to input levels) are different depending on which set of relationships is used. For efficient utilization of resources and cost-minimizing reasons, the use of frontier cost and production relationships in policy formation is very appealing.

FOOTNOTES - CHAPTER I

[1]See F. Balderston [1972], E. Cheit [1971], P. Coombs and J. Hallak [1972], W. Jellema [1971], Chronicle of Higher Education [1970], and V. Smith [1971].

[2]See E. Cheit [1971] for specific cases.

[3]As stated by M. Blaug [1969]:

> The educational system may be conceived of as a kind of processing industry in which certain inputs like teachers, buildings, and equipment are applied for the purpose of processing a raw material (students) into a finished product (again, students). Assuming that the educational process aims at definite objectives, the so-called 'goals of education,' we can inquire whether the inputs are used efficiently to achieve the desired output. In short, we can study the productivity of the educational system like that of any other industry.

A similar statement is made in the introduction to a compendium of papers submitted to the Joint Economic Committee [1969]:

> Institutions of higher education play a vital role in the United States economy. As firms in an industry, these institutions absorb inputs and produce outputs, both of which are of value to the society. The inputs used by institutions of higher education consist not only of the services of their capital facilities and the time and energy of the most highly educated of the Nation's citizens, but also of the time and productive capacity of the students who are in attendance. The outputs of these institutions consist of a more highly educated and productive citizenry, the results of research and the discovery of new knowledge, and, indirectly, a more rapid rate of economic growth.

[4]An example of this type of analysis is outlined in detail in a recent publication by NCHEMS at WICHE [WICHE, 1971].

II. SPECIFICATION OF THE ECONOMIC BEHAVIOR OF HIGHER EDUCATION INSTITUTIONS

The decision makers at institutions of higher education are most likely not guided by the principles of profit maximization or cost minimization. Unfortunately most of the traditional work in economic theory has been geared towards firms that do operate with these types of objectives. However, the basic economic behavior of non-market organizations in general and colleges and universities in particular have received some attention in the literature.[1] Primary areas of concern are the relevance of economic theory to the behavior of these organizations and the many externalities and non-economic factors that influence the behavior of these organizations. These problems are discussed in this chapter and a general framework in which to empirically analyze the frontier production and cost behavior of higher education institutions is developed.

Basic Economic Behavior

One of the fundamental problems in applying economic theory to institutions of higher education is in making the basic behavioral assumptions about the decision-making process of the firm. Several behavioral assumptions for educational firms that run counter to standard economic behavior are listed by H. Levin [1971]:

> (1) the educational managers at all levels lack knowledge of the production set for obtaining particular outcomes.
> (2) substantial management discretion does not exist over which inputs are obtained and how they are organized in educational production.
> (3) little or no competition exists among schools.

(4) prices of both inputs and outputs are not readily available to educational managers.
(5) the incentive or reward structures characteristics of schools seem to have little relation to the declared educational goals of those institutions.
(6) there are no clear signals of success or failure for the schools that are comparable to sales, profits, losses, rates of return, or shares of market.

The above list illustrates the hazards of assuming that many of the basic micro-economic assumptions, such as cost minimization or profit maximization, apply directly to these institutions. Since so little is known about the economic behavior of colleges, an empirical analysis should be formulated with as few behavioral assumptions as possible. By building on a general framework, many of the assumptions can be empirically tested to see if, in fact, they do apply. It is primarily for this reason that the present research starts with the formulation and computation of production relationships and then extends into the analysis of cost relationships. By estimating the production relationships, it is possible to study separately the effects of technical and allocative efficiency. By only analyzing cost this separation is not possible. Due to certain institutional rigidities, an institution may not be able to manipulate its resources in response to relative factor prices and therefore be allocatively efficient. However, if this is the case, it is still informative to measure technical efficiency by itself from the production relationships.

The second area of concern for the application of micro-economic theory to higher education is the influence of externalities and other non-economic factors. Colleges and universities are subject to many external (as well as internal) social and political forces that can have strong effects on their financial and production behavior. In fact, R. Hough [1970] has characterized higher education institutions as "producers of externalities." These institutions must be pollution free, maintain racial and sex balance, and be able to run counter to the general business cycle. Most schools are very dependent on funds from government sources (this dependency is increasing with time) and therefore the schools are influenced heavily by political changes.

It is this complex environment in which these institutions exist that makes an economic analysis of their behavior difficult. Care must be taken to avoid comparing apples and oranges and not to assume that institutions can do things that they cannot possibly do.

Use of Micro-Economics

Having discussed the problems of applying economic theory to higher education, it is helpful to switch to a positive note and see the guidance micro-economics can provide in trying to understand the extreme expenditure and production behavior that is observed across samples of colleges and universities. Since it is apparent that the financial and production behavior of these institutions is quite diverse,[2] an attempt is made to see if economic theory can provide a structure for the examination of these variations. Several possible explanations for this diversity in behavior are listed below.

(1) Unique Institutions. Each school can be assumed to face its own special production possibilities utilizing specialized inputs to produce unique products. This assumes that neither the faculty, students, nor the educational process can be compared across a sample of institutions. This view of the "higher education industry" means

that any cost or production comparison of institutions is futile and costs per student can vary to wide extremes.

(2) Technical Efficiency. It can be assumed that all institutions face the same production possibilities but that they differ in technical efficiency. That is, not all institutions produce a given level and combination of outputs with the minimum amount of some combination of inputs given the technology available to them.

(3) Allocative Efficiency. All schools may face the same productive possibilities and all be technically efficient but some may not be "price" efficient. That is, given the prices of the production inputs some schools may not be using the combination of inputs that leads to the minimum cost of producing a given set of outputs. It is this aspect of the economic behavior of schools that is analyzed in internal pricing studies.[3]

(4) Different Output Preferences. Some schools may prefer a stronger mix of outputs that require more inputs per unit or more expensive inputs. Therefore their average costs per student are higher even though they may be producing at a level that is technically and allocatively efficient. Similarly, some schools may prefer to produce higher quality outputs and therefore have higher costs.

With an appropriate structure, an empirical analysis can provide information as to the relative importance of the above factors in determining the variations in costs per student that are observed for higher education institutions.

Framework for Empirical Analysis

In order to visualize how the present empirical analysis of frontier production and cost relationships fits into a more complete picture of institutional behavior, it is helpful to discuss a fairly generalized institutional decision model as shown in Table 2-1. The optimization problem facing the institution is to maximize some preference function over the level and mix of students, the level and mix of inputs, and the set of institutional characteristics. This maximization is subject to a set of production possibilities between the number of students enrolled, the institutional characteristics, and the input variables. Also the institution faces a budget constraint, part of which is fixed and part is a function of the number of students enrolled and other characteristics of the institution. This type of institutional decision model is developed in much more detail by W. Wagner and G. Weathersby [1971].

Within this model, the present study centers on the empirical description of the production relationships as shown by $F(S, X, C) = 0$ in Table 2-1. An analysis of this function allows one to study the production relationships between students enrolled and the institutional inputs and the ways in which these relationships are affected by different sets of institutional characteristics. It should be noted that it is not necessary to know any of the parameters of the preference function, $U(S, X, C)$, or the revenue function, $G(S, C)$, in order to determine the production relationships, $F(S, X, C)$. Each observed institution represents a point in the multi-dimensional production space and all the institutions together, with their diverse behavior, describe the observed production possibility set. Without

TABLE 2-1

Institutional Decision Model

maximize $U(S, X, C)$

subject to:

$$F(S, X, C) = 0$$

$$W'X \leq B_0 + G(S, C)$$

$$X \geq 0$$

$$S \geq 0$$

where:

S = a vector of student enrollments;

X = a vector of institutional labor and capital inputs;

C = a vector of institutional characteristics (e.g., quality, program mix, scale, . . .);

W = a vector of input prices;

B_0 = that part of the institutional budget which is not based on any of the enrollment or characteristic variables;

$U(S, X, C)$ = the institutional preference function over the level and mix of students, the level and mix of inputs, and the institutional characteristics;

$F(S, X, C)$ = the implicit production relationship between students enrolled and institutional inputs and characteristics;

$G(S, C)$ = a revenue function giving that part of the budget which is directly related to enrollment and other characteristics of the institution.

additional information about the parameters of an institution's preference and revenue functions, however, it is not possible to determine why an institution is observed at one point in the production space rather than at another point. Total institutional costs are equal to the summation of each input times its unit price (shown in Table 2-1 as W'X), and therefore an analysis of total institutional costs is tied directly into the production relationships through the level and mix of the input variables.

In order to estimate or compute the relationships that determine the implicit function denoted by F(S, X, C), some assumptions are necessary. First, it is assumed that the production process may be joint.[4] This characterization of production behavior assumes that all of the types of students are enrolled into a single process which may not be additively separable into sub-processes for each type of enrollment and that the total amount of each input used (and, hence, total costs) cannot be directly allocated to the respective types of students. This type of behavior means that the production process cannot be broken down and must be studied at the institutional level. With this approach the production model has to express the technological relationships between the total amounts for each input and each enrollment type. A joint production process is in contrast to separable production processes. The latter method of characterizing production behavior is based on the assumptions that each output is produced by a separate process, that each of these processes can be identified, and that each input can be allocated to each process or output. Application of this approach to the higher education industry requires the development of separate production relationships for each enrollment category (undergraduates, graduates, and specialized) and the allocation of each input (faculty, secretaries, classroom space, . . .) to each of these processes.

A second assumption is that the production relationships are convex. This assumption is almost always made in economic theory and it amounts to assuming that if two or more points are attainable in practice, then so is any point representing a weighted average of the two points.[5] The importance of this assumption is illustrated later in the development of the computational method used in the analysis.

The third assumption is that variables measuring characteristics of the institutions besides inputs and students enrolled can be included into the specification of the production possibility set. By treating all of the characteristic variables as measures, in some sense, of production quality, the approach discussed by S. Dano [1966] can be used directly. He included quality parameters into the production relationships in the following manner:

> Some kinds of quality change - particularly those concerning non-quantitative quality criteria - can be effected only by discontinuous change in the technology, that is, by switching to a different production relationship. On the other hand, as to such dimensions as are quantifiable, it seems plausible to assume that a continuous range of quality levels - as represented by the values of the continuous quality parameters - can be produced within the same basic technology; the same inputs are used but higher product quality, like a higher rate of output, requires more of some or all inputs.

As shown in Table 2-1, a whole vector of institutional characteristic variables, C, are included in the production relationship, $F(S, X, C)$. A detailed discussion of these characteristic variables and how they relate to the input and enrollment variables is given in the following chapter of this paper. In addition, the institutional categories (public and private universities, etc.) are treated as "non-quantitative quality criteria," so separate production relationships are estimated for each group of institutions.

For the purposes of this study, the function F(S, X, C) is defined so as to yield the <u>minimum</u> level of inputs for given levels and mixes of enrollment and a given set of institutional characteristics.[6] This type of relationship is necessary in order for the institution's preference function to be maximized. This implies that the "average" relationships between all of these variables are <u>not</u> desired but that the "frontier" relationships as observed from the data are needed. The distinction between the average and the frontier relationships has a large effect on the estimation procedure to be used and the problems this poses are discussed in more detail in the estimation chapter of this paper.

It is in the economic framework described above that the empirical analysis of this study is performed. Relatively few behavioral assumptions are made, since it is not being assumed that colleges and universities are cost minimizers. In fact, one output of this research is a measure of relative production efficiency for institutions within homogeneous samples.

FOOTNOTES - CHAPTER II

[1] Collections of papers on this general subject can be found in M. Blaug [1969] and the compendium of papers [Joint Economic Committee, 1969]. Two other selected works are W. Bowen [1968] and M. Feldstein [1968].

[2] See the empirical studies by H. Jenny and H. Wynn [1970, 1972], the Columbia Research Associates [1971], and J. Powel and R. Lamson [1972].

[3] See the collection of papers by D. Breneman [1971].

[4] Articles on this subject include W. Diewert [1968], Y. Mundlak [1963], R. Pfouts [1961], and H. Vinod [1968].

[5] In a recent article, G. Hanoch and M. Rothschild [1972] propose methods of testing the validity of the convexity assumption from data on inputs and outputs. Unfortunately, use of their methods requires the data to consist of observations on competitive profit maximizing firms.

[6] This follows straight from the standard textbook definition of a production function; for example, see J. Henderson and R. Quandt [1958].

III. THE DATA

The amount of data required for the empirical analysis formulated in the previous sections is quite large. Several sources of institutional data are tapped in order to perform the analysis on a large cross-section of higher education institutions for the 1968 fiscal year. The data sources are described below and the constructed variables are defined. The sample sizes and other characteristics of the data are also presented in this chapter.

Data Sources

The primary data source for this research is that collected and assembled by the Department of Health, Education and Welfare--Office of Education on their Higher Education General Information Surveys (HEGIS). These surveys are sent annually to all institutions of higher education in the United States and the response rate is very high. The HEGIS data is currently the most extensive nationwide data base in existence for colleges and universities. The usual problems of reporting errors and inconsistencies resulting from different accounting practices and from different interpretations of the HEGIS data forms are undoubtedly present. However, the procedures illustrated in Chapter VI with respect to the sensitivity of the empirical results to the data for individual institutions provide ways of screening the data for "bad" observations.

Five of the HEGIS data files for the fiscal year 1968 are used extensively. The survey titles as well as brief outlines of the data on each questionnaire are listed below.

(1) Financial Statistics of Institutions of Higher Education. The data reported in this survey are a detailed description of the revenues by source and the expenditures by function for each institution. The survey is comparable to a balance sheet and income statement for each college and a detailed description of each item in the survey is given in College and University Business Administration (American Council on Education, [1968]).

(2) Comprehensive Report on Enrollment. This survey presents an extensive picture of the number and mix of different types of students enrolled at each institution.

(3) Employees in Institutions of Higher Education. This survey is very detailed, so it is only outlined here. The first part of the survey gives the number of part-time and full-time personnel by primary function (instruction, organized research, library, extension, and administration). The second part lists the number of faculty by academic rank and major area of teaching. Part three shows the salary distribution by academic rank, and part four lists the salaries of selected administrators. The final section gives the distribution of highest educational level achieved for each academic rank.

(4) Inventory of College and University Physical Facilities. The data in this survey are the total number of square feet of classrooms, laboratories, offices and study rooms assignable to specific functions (instruction, organized research, public service, and general administration).

(5) Degrees and Other Formal Awards. Part one of this survey gives the number of first professional degrees conferred in selected

fields. The second part lists the number of Bachelor, Master, and Doctoral degrees conferred by major field of study. The final part shows the number of degrees and completions based on less than four years of work beyond high school.

To supplement the HEGIS data, information from three additional sources is included into the data set. The first is the Carnegie Commission's classification of higher education institutions which is described in detail in their publication, New Students, New Places (Carnegie Commission, [1971]). The second item of data is the Gourman Quality Rating of colleges and universities constructed by J. Gourman [1968] and described later in this chapter. The final pieces of data are rough estimates of per unit costs of building space generated by Bowen and Douglass [1971].

A serious data problem exists for the multi-campus systems, since the financial, employee, and physical facility data are usually reported on a total system basis, and enrollment data, the earned degree data, and the Gourman quality rating are available for each individual campus. This reporting behavior means that any financial or production analysis must treat these schools as complete systems. The problem with this approach is that many of these systems have branches that are universities, two-year colleges, or other primarily undergraduate institutions, and putting them all together defeats the purpose of trying to construct homogeneous samples. In addition, the fact that these institutions are organized as systems implies a different structure of organization and therefore they should be studied separately. In view of these problems, the multi-campus systems that report financial, employee and physical facility data as a system are deleted from the empirical analysis.

Variable Definitions

All of the variables used in this study are defined in Table 3-1 along with mnemonics that are used throughout the rest of the paper. The institutional variables are divided into three groups: input variables, enrollment variables, and characteristic variables. These variables are discussed by category in the remainder of this section. The selection of this particular set of variables is the result of attempting to reach a balance between capturing the relevant dimensions of institutional behavior that affect costs and limiting the number of variables so the computations are manageable. Given the data, more detail could be included within each variable category but the marginal increase in information to be gained did not appear to warrant the extra problems and expense in doing the computations.

For higher education institutions, the primary production inputs are: labor in the form of junior and senior faculty, non-professional departmental employees, general administration, library, and other support employees; capital in the form of classrooms, laboratories, departmental offices, libraries, and general administration offices; and equipment, supplies, and materials. The relative size of each of these major groups of inputs (labor, facilities, and supplies) has been estimated by Bowen and Douglass [1971] and their estimates are shown in Table 3-2 for three different types of instruction. Extensive data is utilized in the present study on employees and physical facilities of institutions of higher education but no data is currently available on the equipment, supplies, and materials used by these institutions. Since the percentage of total

TABLE 3-1

Variable Descriptions

Institutional Input Variables

1) SFAC - senior faculty - resident instruction and departmental research, professional personnel, senior staff (includes deans, department heads, and all others whose primary function is resident instruction and departmental research).

2) JFAC - junior faculty - resident instruction and departmental research, professional personnel, junior staff (teaching and departmental research assistants).

3) NPDP - resident instruction and departmental research, non-professional personnel (clerks, secretaries, stock-room attendants).

4) GALB - all personnel in general administration, general institutional and student personnel services, organized activities relating to instructional departments, physical plant maintenance, and all library personnel.

5) CLSP - departments of instruction and research, total square feet assignable of classroom space.

6) LASP - departments of instruction and research, total square feet assignable of laboratory space.

7) GASP - total square feet assignable of departmental office, departmental study, general administration office, and library space.

8) COST - total educational and general expenditures which includes the cost of instruction and departmental research, extension and public service, libraries, physical plant maintenance and operation, general administration, general institutional and student services, organized activities relating to educational departments, organized research, and other sponsored programs.

Student Enrollment Variables

1) UGPT - part-time undergraduates in programs wholly or chiefly creditable towards a bachelor's degree.

2) UGFT - full-time undergraduates in programs wholly or chiefly creditable towards a bachelor's degree.

3) GRAD - all students who are candidates for a master's or higher degree.

4) OTHE - all students enrolled in a professional school or program which requires at least two or more academic years of college work for entrance and a total of at least six years for a degree, all students in organized occupational programs of less than four years and not chiefly creditable toward a bachelor's degree, and extension students or students who do not take their college work on the regular campus.

TABLE 3-1 (continued)

Institutional Characteristic Variables

1) GOUR - the Gourman institutional quality rating. The arithmetic mean of a departmental and a non-departmental rating by J. Gourman [1968]. The departmental rating is a rating of the academic departments in terms of such things as accreditation and the proportion of students receiving scholarships and fellowships. The second component is a rating of non-departmental aspects of the institution such as the administration's "commitment to excellence," the level of financial aid available to students, and faculty morale.

2) PSCI - percent laboratory science and engineering degrees of total degrees granted.

3) RETN - upper division enrollment as a percent of first-time enrollment for the previous two years.

4) SCLE - total current fund revenues - ($1,000).

5) NFLD - number of fields granting degrees (Note: a B.S. in Biology, a M.S. in Biology, and a Ph.D. in Biology count as three fields).

6) GRTH - growth rate of total enrollment, 1965-70 (growth rate = β/α as estimated from the regression $\text{enrollment}_t = \alpha + \beta t$, $t = 1, \ldots, 6$).

7) PRES - percent research revenues of total revenues.

8) PEXT - percent extension and public service expenditures of total education and general expenditures.

Institutional Categories

1) UNIV-PUB: public doctoral-granting institutions with emphasis on research.

2) UNIV-PRI: private doctoral-granting institutions with emphasis on research.

3) COMP-PUB: public comprehensive colleges that offer a liberal arts program as well as several other programs.

4) COMP-PRI: private comprehensive colleges that offer a liberal arts program as well as several other programs.

5) LIMC-PUB: public limited comprehensive colleges that offer a liberal arts program as well as at least one professional or occupational program.

6) LIBA-PRI: private highly selective liberal arts colleges.

7) OLBA-PRI: private less selective liberal arts colleges.

TABLE 3-2

Estimated Cost Per Class and Percent Each Component Cost is of Total Cost

Component	Lab. Science Class		Other Science Class		Non-Studio Fine Arts Class	
Salaries	$4,815	74.5%	$3,871	84.3%	$3,115	88.7%
Facilities	1,244	19.3	541	11.8	321	9.1
Equipment	400	6.2	180	3.9	77	2.2
Total	6,459	100.0	4,592	100.0	3,513	100.0

SOURCE: Bowen and Douglass [1971], pp. 50-51.

expenditures per class that is due to supplies is very small (2% to 6%), this should not be a very serious omission in the sense that it should not affect the production relationships with respect to the other input variables.

For the purposes of this research, student enrollment is separated into four components: full-time undergraduates, part-time undergraduates, graduate students, and specialized students (those enrolled in extension, first professional, or occupational programs). The latter group of students is a residual of enrollment in the standard baccalaureate and graduate programs at 4-year colleges and universities and this residual is their primary common characteristic. In order to keep the number of variables at a reasonable level, total headcount of graduate students and specialized students is used rather than including part-time and full-time separately. Except for the university samples, the relative magnitudes of graduate and specialized students enrolled are very small. Even if graduate and specialized enrollment were broken down into full-time and part-time, it is doubtful that it would be possible to identify different production and cost relationships between them. For the public university sample, some calculations are performed using graduate part-time and graduate full-time as separate variables. The results from these calculations are so close to the results generated with just one graduate enrollment variable that it does not seem necessary to continue with two graduate variables for any of the institutional categories. The relationships between undergraduate, graduate, and other programs can be analyzed with the four variables listed in Table 3-1, and the behavior of part-time versus full-time undergraduates can also be studied. A more complete analysis would include a much finer breakdown of enrollment, but the computations would be much more cumbersome.

The institutional characteristic variables as listed in Table 3-1 measure various aspects of the institution's behavior that are not captured in either the input or the enrollment variables. It is assumed that these variables have effects on the production and cost relationships between the institutional inputs and the enrollment variables. In order to include these variables into the analysis, it is necessary to specify the direction of the effect of each of these variables on the production relationships. If the direction is specified incorrectly, the computed relationships will indicate that the variable does not affect cost and production behavior. That is, if the variables are specified wrong, the constraints in the linear programming model (developed in Chapter IV) will never be binding, and therefore changing the value of the variable will not change the solution. Since the empirical results show all of the characteristic variables to behave as expected, none of the specifications appear to be incorrect.

Six of the characteristic variables (GOUR, PSCI, RETN, NFLD, PRES, and PEXT) are assumed to behave like the enrollment variables. That is, it is assumed that a higher value for each of these variables requires more institutional inputs for a given level of enrollment or decreased enrollments for a given input level. An institution with a higher quality rating, a larger percentage of science-oriented programs, a higher enrollment retention rate, a larger number of fields granting degrees, or a larger commitment to research or public service is assumed to require more inputs than a comparable institution with a lower value for one or any combination of these six variables. It should be noted that the retention variable (RETN) is not net of transfer students. That is, an institution could have a high drop-out rate and a high inflow of upper division transfer students and still have a fairly high value for RETN. This clouds the interpretation of the variable somewhat, but it should be noted that a high value for RETN still means a high proportion of upper division to lower division students. Also, additional resources are spent in advising and counseling if the transfer rate is low or in processing and advising transfer students if the transfer rate is high.

The remaining characteristic variables, SCLE and GRTH, are assumed to behave like inputs in that a higher value for each of these variables results in fewer inputs for a given enrollment or an increase in enrollment for a given level of inputs. This assumes, in effect, that higher education institutions may exhibit increasing returns to scale. Several arguments can be presented to indicate that the enrollment growth variable (GRTH) could be treated either as an input or as an enrollment variable. It is hypothesized here that institutions experiencing rapid enrollment growth are able to better utilize their resources than no- or slow-growth institutions. H. Jenny and G. Wynn [1970] show an inverse relationship between enrollment growth and expenditures per FTE for their sample of private liberal arts colleges. The empirical results presented later in this study also appear to verify this hypothesis.

The final category of variables listed in Table 3-1 is the classification of institutions used in this study. The classification is essentially a condensed version of the Carnegie Commission's classification (Carnegie Commission, [1971]). This variable is used solely as a means of separating the total sample of institutions into groups of colleges and universities with relatively homogeneous goals and missions. Institutions that are not included in this study and are not in the groups listed in Table 3-1 are the professional schools and other specialized institutions (theological seminaries, medical schools, schools of engineering, schools of business, schools of art, music, design, schools of law, and teachers colleges) and the two-year colleges and institutes.

Sample Size and Diversity of Behavior

Due to the large number of variables constructed from the data of several surveys that are included in this analysis, the sample of institutions being studied is roughly fifty percent of the total number of four-year colleges and universities in the United States. This is not a random sample but a sample of all the institutions that have reported the necessary data. Table 3-3 shows the representation of institutions in the sample by type of institution. Due to the problem of multi-campus systems discussed earlier, the public universities are the least represented group.

To illustrate the variation in the magnitudes across institutional categories of all the variables defined in Table 3-1, the mean of each variable is given in Table 3-4 for each institutional category. As a further illustration of the diverse behavior exhibited by the data, the range for each of the characteristic variables is given in Table 3-5 for each group of institutions. These wide ranges indicate substantial differences in the characteristics of institutions within groups generally thought to be fairly homogeneous. The empirical analysis of this study relates the variations in these characteristics to the production relationships between institutional input and enrollment levels.

TABLE 3-3

Sample Representation of Higher Education Institutions

Institutional Type	Total Number in U.S.*	Number in Sample	Percent of Total in Sample
Public Universities	101	37	36.6
Private Universities	63	31	49.2
Public Comprehensive Colleges	210	105	50.0
Private Comprehensive Colleges	147	91	62.0
Public Limited Comprehensive Colleges	133	59	44.4
Private Highly Selective Liberal Arts Colleges	121	81	67.0
Private Less Selective Liberal Arts Colleges	555	269	48.5
Total	1,330	673	50.6

*As reported by the Office of Education.

TABLE 3-4

Variable Means by Institutional Category

Variables	Public Universities	Private Universities	Public Comprehensive Colleges	Private Comprehensive Colleges	Public Limited Comprehensive Colleges	Private Highly Selective Liberal Arts Colleges	Private Less Selective Liberal Arts Colleges
Gourman Quality Rating	453	546	366	379	344	422	343
Percent Science Degrees	21	25	12	15	8	15	10
Enrollment Retention	97	98	80	85	68	76	71
Institutional Scale	4,650	5,034	903	592	436	431	206
Number of Fields	125	90	42	30	23	22	17
Enrollment Growth Rate	7.6	3.9	10.0	3.0	10.6	2.5	2.9
Percent Research Revenues	15.0	28.4	1.6	1.8	0.9	1.2	0.5
Percent Public Service Expenditures	5.4	1.5	1.4	1.1	0.8	0.8	0.7
Part-time Undergraduates	1,041	563	615	461	141	24	69
Full-time Undergraduates	9,539	4,073	4,335	2,136	2,203	1,114	822
Graduate Students	2,697	2,565	521	397	175	30	20
Specialized Enrollment	498	678	158	119	13	5	4
Senior Faculty	778	670	258	157	131	96	61
Junior Faculty	178	143	17	8	3	2	1
Nonprofessional Departmental Personnel	276	180	47	25	24	10	8
General Administration Personnel	1,070	1,209	177	146	97	122	56
Classroom Space	150,811	94,190	70,056	77,156	41,746	29,325	22,525
Laboratory Space	387,278	211,831	87,310	47,533	40,311	33,431	18,311
Administration Office Space	406,179	298,719	101,546	87,762	53,818	56,569	31,350

SOURCE: Office of Education - HEGIS institutional data, fiscal 1968.

TABLE 3-5

Statistics for the Characteristic Variables by Institutional Category

Type	Statistic	Gourman Quality Rating	Percent Science Degrees	Enrollment Retention	Institutional Scaler	Number of Fields	Enrollment Growth Rate	Percent Research Revenues	Percent Public Service Expenditures
Public Universities	Minimum	307	4	57	1,214	61	-2.0	0.0	0.0
	Average	453	21	97	4,650	125	7.6	15.0	5.4
	Maximum	749	47	180	22,441	225	37.0	38.0	28.0
Private Universities	Minimum	354	4	39	494	24	-4.0	0.0	0.0
	Average	546	25	98	5,034	90	3.9	28.4	1.5
	Maximum	772	90	149	24,596	224	72.0	94.0	30.0
Public Comprehensive Colleges	Minimum	284	1	17	180	10	-1.0	0.0	0.0
	Average	366	12	80	903	42	10.0	1.6	1.4
	Maximum	449	44	430	3,991	101	37.0	24.0	28.0
Private Comprehensive Colleges	Minimum	245	0	35	145	2	-6.0	0.0	0.0
	Average	379	15	85	592	30	3.0	1.8	1.1
	Maximum	548	64	436	2,101	68	35.0	30.0	25.0
Public Limited Comprehensive Colleges	Minimum	260	0	26	123	3	-3.0	0.0	0.0
	Average	344	8	68	436	23	10.6	0.9	0.8
	Maximum	411	51	158	1,443	63	29.0	25.0	16.0
Private Highly Selective Liberal Arts Colleges	Minimum	298	0	42	109	4	-7.0	0.0	0.0
	Average	422	15	76	431	22	2.5	1.2	0.8
	Maximum	600	81	108	1,299	46	22.0	11.0	9.0
Private Less Selective Liberal Arts Colleges	Minimum	247	0	27	7	1	-9.0	0.0	0.0
	Average	343	10	71	206	17	2.9	0.5	0.7
	Maximum	433	53	145	543	45	66.0	35.0	21.0

SOURCE: Office of Education - HEGIS institutional data, fiscal 1968.

IV. ESTIMATION TECHNIQUES

In formulating a method of estimating the relationships between the enrollment variables, the institutional inputs, and the institutional characteristic variables, several problems must be confronted. First as discussed earlier, it is desirable to estimate the frontier relationships between all of these variables rather than the average relationships. Second, it is difficult to select a single variable as a dependent variable that is to be "explained" as a function of all the other variables. An estimation method is needed that does not require one variable to be selected as being caused by all the others. The decision process that determines the levels of all these variables is very complex, and no one single variable is unique, in that it is determined once all the other variables are set at particular levels. Third, since so very little is known about the technology of the higher education industry, it is difficult to provide *a priori* reasoning about the shape of the frontier relationships between the enrollment variables and the institutional input and characteristic variables. That is, it is hard to select one particular functional form as being more appropriate to estimate than any other function. Initial attempts at fitting different regression equations to the data indicate that it would be extremely difficult to statistically discriminate between alternative models on the basis of the cross-sectional data.

In the following two sections of this chapter, a computational method is described that is free from the above problems and is used extensively in this study as a means of generating the frontier relationships between enrollment variables, institutional input and characteristic variables. The fourth section of this chapter is a comparison of this "less-known" computational method with the more "well-known" method of least-squares regression. In the final section, a modification of the basic computational procedure used in this study is discussed which allows the calculation of frontier cost relationships for alternative enrollment mixes and levels and different specifications of institutional characteristics.

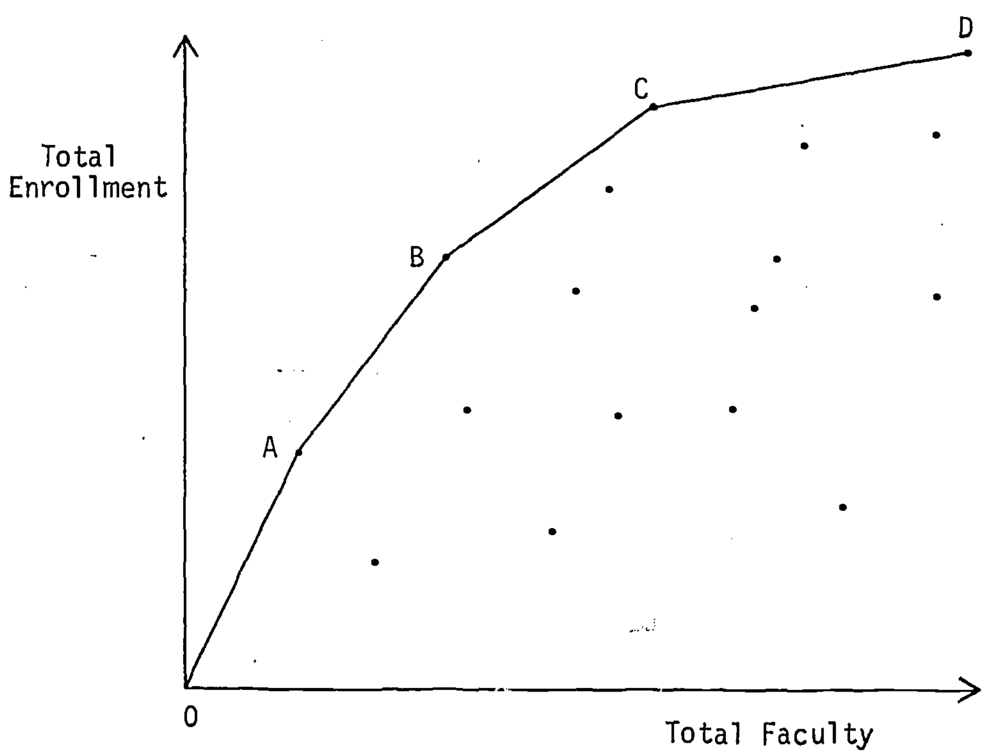
A Graphic Approach

If the empirical analysis consists of only two variables (i.e., total enrollment and total faculty), the best procedure for generating the frontier relationship between these two variables is to simply plot the observations as shown in Figure 4-1. Each plotted point represents an institution, and the frontier relationship between total enrollment and total faculty is given by the curve OABCD. That is, points on this curve represent the maximum total enrollment observed for a given of total faculty or, alternatively, the minimum number of faculty observed for a given level of total enrollment. Note that in order to make the above statement, it is necessary to assume that the production relationship is convex. This assumption states that if two points are attainable in practice (for example, B and C above), then so is any point representing a weighted average of them (points on the line connecting B and C).

The frontier relationships between different inputs (i.e., junior faculty versus senior faculty) and alternative enrollments (i.e., undergraduates versus graduates) are also to be determined, so a

FIGURE 4-1

Graphic Approach - Two Variable Example



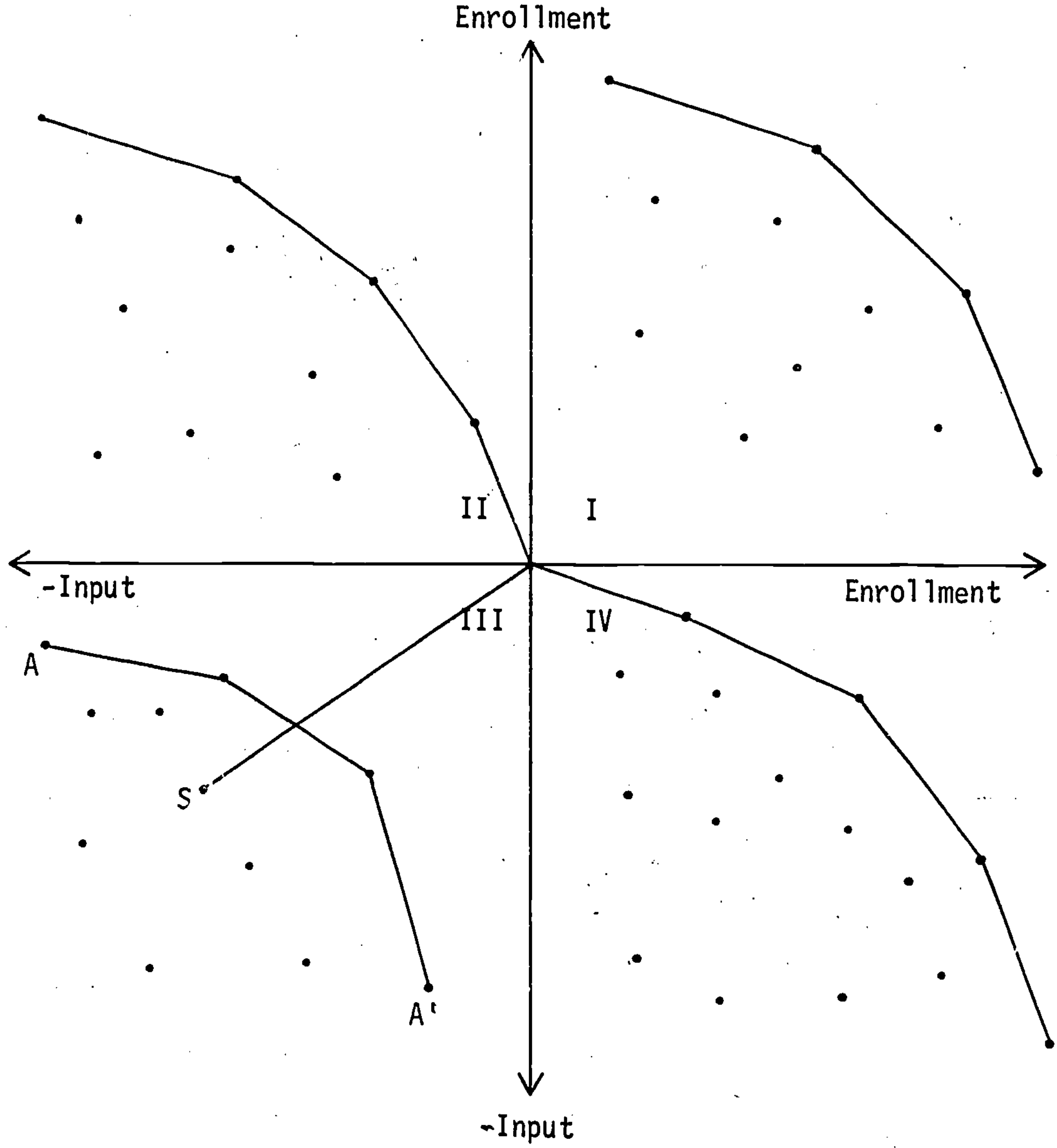
consistent way of constructing the frontier surface is needed. In order to accomplish this graphically as well as computationally, it is necessary to treat the input variables (and any characteristic variables specified to behave like input variables) as negative and the enrollment variables (and any characteristic variables assumed to behave like enrollment variables) as positive. These relationships are illustrated in Figure 4-2. Note that for all the relationships, enrollment versus enrollment, enrollment versus input, and input versus input, the desired frontier curve is the northeast portion of the outer ring circumscribing the scatter of points. The familiar transformation curve between two "outputs" of a production process appears in quadrant I of the graph in Figure 4-2. Productivity curves are shown in quadrants II and IV (these are always identical) and an input isoquant curve appears in the third quadrant.

If additional variables are added to this two-variable analysis, the graphic approach soon becomes more difficult and less useful. A possible approach might be to group the institutions with respect to all of the variables except two and then to plot separate diagrams showing the relationship between the two excluded variables. If this is extended to an analysis of 15 or 20 variables, the sample size would have to be very large to permit such a detailed analysis to be done.

The "Farrell" Approach

For the case of many variables, a computational method originated by M. J. Farrell [1957, 1962] provides an efficient procedure for generating relationships like those shown in Figure 4-2. Essentially, Farrell's method is to plot the observations as points in a space of

FIGURE 4-2
Illustration of Graphic Approach



as many dimensions as there are variables included in the analysis, to form the convex hull of this set of points, and to take the appropriate part of the surface of the convex hull as the estimate of the frontier relationship between all of the variables.

In order to determine the frontier relationships with Farrell's basic approach, J. Boles [1971, 1972] has greatly simplified the computations needed by formulating the procedure in terms of a linear programming problem. The link between the graphic approach illustrated above and the linear programming approach can best be made for the case of one enrollment variable (i.e., full-time undergraduates) and two inputs (i.e., junior faculty and senior faculty). The desired relationship is illustrated in the third quadrant of Figure 4-2. Note that in order for the input isoquant to have any meaning, the two input variables should be scaled by the enrollment variable. The relationship between the two input variables with all the other variables held constant is desired. In order to locate the observed institutions that determine the frontier relationship between the two input variables, each scaled by enrollment, the procedure is to express the coordinates of each institution s as a linear function of the coordinates of the other institutions that lie <u>closest</u> to the origin of the graph in Figure 4-2. That is, find two institutions (a and b) such that:

$$z_a \left(\frac{JFAC_a}{UGFT_a}\right) + z_b \left(\frac{JFAC_b}{UGFT_b}\right) = \frac{JFAC_s}{UGFT_s}$$

$$z_a \left(\frac{SFAC_a}{UGFT_a}\right) + z_b \left(\frac{SFAC_b}{UGFT_b}\right) = \frac{SFAC_s}{UGFT_s}$$

and $(z_a + z_b)$ is a maximum over all possible pairs of institutions a and b . The two institutions that satisfy the above maximization problem lie on curve AA'. If institution s lies on the curve, the solution to the above problem is $z_s = 1.0$ with the rest of the z's equal to zero.

By defining the variables,

X_{it} = the quantity of the i^{th} input used by the t^{th} institution

Y_{jt} = the quantity of type j students enrolled at the t^{th} institution

the above maximization problem for T institutions can be written in a linear programming framework as:

$$\text{Maximize} \quad \sum_{t=1}^{T} z_t$$

$$\text{Subject to:} \quad \sum_{t=1}^{T} z_t \left(\frac{X_{it}}{Y_{rt}}\right) \leq \frac{X_{is}}{Y_{rs}} \qquad i = 1,2$$

$$z_t \geq 0 \qquad t = 1,\ldots,T$$

This simple, three-variable model can be generalized to include several input variables, other enrollment variables, and the institutional characteristic variables. Let

C_{kt} = the k^{th} characteristic variable for the t^{th} institution,

then the general linear programming model for T institutions, N input variables, M enrollment variables, and Q characteristic variables is written as:

$$\text{Maximize} \quad \sum_{t=1}^{T} z_t$$

$$\text{Subject to: } \sum_{t=1}^{T} z_t \left(\frac{X_{it}}{Y_{rt}}\right) \le \left(\frac{X_{is}}{Y_{rs}}\right) \qquad i = 1,\ldots,N$$

$$\sum_{t=1}^{T} z_t \left(\frac{Y_{jt}}{Y_{rt}}\right) \ge \left(\frac{Y_{js}}{Y_{rs}}\right) \qquad j = 1,\ldots,\ r-1,\ r+1,\ldots,M$$

$$\left(\sum_{t=1}^{T} z_t\ C_{kt}\right) \Big/ \sum_{t=1}^{T} z_t \le C_{ks} \qquad k = 1,\ldots,Q_1$$

$$\left(\sum_{t=1}^{T} z_t\ C_{kt}\right) \Big/ \sum_{t=1}^{T} z_t \ge C_{ks} \qquad k = Q_1+1,\ldots,Q$$

$$z_t \ge 0 \qquad t = 1,\ldots,T$$

The input constraints and the enrollment constraints are identical except that the inequality sign is reversed. This reversal is consistent with the differences in sign used in the graphic illustration shown in Figure 4-2. The constraints for the characteristic variables are considerably different from the input and enrollment variable constraints. The weighted average form of these characteristic variable constraints is necessary in order to account for the differences in magnitude of the input and enrollment variables across the sample of institutions. The characteristic variables are constructed so that their magnitudes are independent of the size of the institution (unlike the input and enrollment variables). To illustrate, consider institution A with an enrollment of 1,000, a faculty of 50, and a quality rating of 400. To construct an institution twice as large as A but with all the same characteristics, all the input and enrollment variables should be doubled. The characteristic variables, however, should not change. It is for this reason that the characteristic variables are not scaled by the enrollment variable in the linear programming formulation given above. Similarly, since the numerator, $\sum_{t=1}^{T} z_t$, does not necessarily sum to 1.0, it is necessary to divide $\sum_{t=1}^{T} z_t C_{kt}$ by $\sum_{t=1}^{T} z_t$. Note that this treatment is not necessary in formulating the conceptual procedure of forming the convex hull of the observations in (N + M + Q - 1) space but that the special form of the constraints is needed for the linear programming formulation of the problem.

The actual linear programming model used in this study is a slight variant of the model outlined above, and it is given in Table 4-1. The only changes are that each column has been multiplied by Y_{rt} and that the characteristic variable constraints are written in a form consistent with the other constraints. With the computational procedure discussed above, it is possible to determine which institutions describe the frontier relationships between all of the variables and also to determine how far the other institutions are from the frontier surface. It should be noted that the choice of the variable appearing in the linear programming objective function (Y_{rt} in the above example) depends on the information that is desired. In order to determine which institutions are on the frontier surface, any of the variables can be used in the objective function. If information about the <u>shape</u> of the production surface is desired, the choice of objective function variable is more important. The distance being maximized (or minimized in the case of an input variable) is perpendicular to the axis of the variable in the objective function. It should be stressed that this is <u>not</u> comparable to the choice of dependent variable in a regression equation, where the results can be

TABLE 4-1

The LP Computational Model

Maximize $\sum_{t=1}^{T} z_t Y_{rt}$

Subject to: $\sum_{t=1}^{T} z_t X_{it} \leq X_{is}$ $\quad i = 1,\ldots,N$

$\sum_{t=1}^{T} z_t Y_{jt} \geq Y_{js}$ $\quad j = 1,\ldots,r-1,\ r+1,\ldots,M$

$\sum_{t=1}^{T} z_t Y_{rt} (C_{kt}-C_{ks}) \leq 0$ $\quad k = 1,\ldots,Q_1$

$\sum_{t=1}^{T} z_t Y_{rt} (C_{kt}-C_{ks}) \geq 0$ $\quad k = Q_1+1,\ldots,Q$

$z_t \geq 0$ $\quad t = 1,\ldots,T$

drastically different depending on the variable selected. With this linear programming approach, the results are always consistent no matter in which direction towards the frontier surface the results are generated.

In addition to using observed institutions on the right-hand side of the constraints, hypothetical institutions can be constructed and used in the analysis as well. This procedure makes it possible to analyze more systematically the frontier relationships between different variables.

Comparison of Estimation Methods

Having discussed in detail the Farrell method of determining the frontier production relationships, it is helpful to compare his approach to least-squares regression and constrained-residuals regression methods of estimating production and cost relationships. Constrained-residuals regression is the method where the regression residuals are all constrained to have the same sign. This constraint forces the estimated function to the "frontier" of the observations. This procedure was originally suggested by Aigner and Chu [1968] and has been implemented by C. Timmer [1971] and H. Levin [1971]. Several aspects of these three estimation methods are discussed below with respect to their differences, advantages, and disadvantages. References to two other empirical studies using the Farrell approach are W. D. Seitz [1971] and B. L. Sitorus [1966], and some further developments of the approach are found in W. D. Seitz [1970].

Error Specification: Least-squares regression assumes that the error term in the model is distributed from $-\infty$ to $+\infty$ and that the error is usually attributed to noise in the model. For both constrained-residuals regression and Farrell's method, the error term is distributed from 0 to $+\infty$ (assuming actual output is subtracted from the frontier level of output) and is attributed to differences in efficiency across the sample of observations. These methods, therefore, present alternatives at the extremes. Least-squares appropriately takes into account noise but ignores efficiency differences (or at best assumes that efficiency is a component of the noise). The other two estimation methods ignore noise (or at best assume it is small relative to efficiency variations) and include a measure of efficiency. The method of least-squares estimates the <u>average</u> production relationships, which have economic meaning only if all the "firms" in the sample are cost-minimizers (or profit-maximizers), while constrained-residuals regression and Farrell's method estimate the <u>frontier</u> production relationships as actually defined in economic theory.

<u>Functional Form</u>: Both least-squares and constrained-residuals regression require a prior specification of the functional relationship between the inputs and outputs. For the one output, N input case many alternative functions are discussed and tested in the literature (see J. Ramsey and P. Zarembka, [1971]). With least-squares, goodness-of-fit statistics can be used to let the data help in selecting one functional form over several alternatives. For constrained-residuals regression, the sampling theory has not been developed, so choice of a functional form must be done primarily on the basis of prior information and judgment. In addition, the statistical problems of multicollinearity and heteroscedasticity

caused by the combining of small and large institutions in the sample make the functional form problem even more difficult for both regression methods.

Several empirical studies in higher education (R. Radner, [1971]; R. Gough, [1970]; D. Breneman, [1970]; H. Levin, [1971]; J. Maynard, [1971]; and L. Perl, [1971]) have used regression techniques, and the functional forms estimated in these studies are summarized in Table 4-2. None of the studies presented results for alternative functional specifications.

Farrell's approach does not require the specification of a functional relationship between the outputs and inputs. Except for the assumptions of convexity, non-positive slopes for isoquants and transformation curves, and non-negative slopes for productivity curves, no functional form restrictions are placed on the data.

Multiple Products: For least-squares and constrained-residuals regression, the usual procedure for the case of a single output is to regress the output on all the inputs. For the case of more than one output, either one output has to be arbitrarily chosen as the dependent variable and the others considered as independent variables, or an output index has to be constructed to reduce the vector of outputs to a single variable. In most higher education empirical work, the latter approach is heavily used. Various weighting schemes are constructed in order to compute an "adjusted FTE enrollment" figure which supposedly reflects the differences in part-time/full-time and undergraduate/graduate students. In his study of graduate education, D. Breneman [1970] used the first approach and placed the number of Ph.D.'s as the dependent variable and let the number of

TABLE 4-2

Comparison of Regression Specifications

Study	Unit of Analysis	Dependent Variable	Functional Form
R. Radner [1971]	Institution	FTE Faculty	Linear; quadratic in the number of students
R. Hough [1970]	Institution	Degrees Granted	Log-linear
D. Breneman [1970]	Department	Ph.D.'s Produced	Linear; all variables scaled by graduate enrollment
H. Levin [1971]	Student	Verbal Score	Linear
J. Maynard [1971]	Institution	Cost per Student	Quadratic in the number of students
L. Perl [1971]	Student	Dummy Variable*	Linear; quadratic in the number of students

*Equals one if the student graduates and equals zero otherwise.

Master's students be one of the explanatory variables. The problem with this procedure is that if the two variables were switched around, the results could be drastically different. A third alternative is available for the least-squares approach. Canonical correlation may be used in order to regress a linear combination of outputs on the inputs. The analogy of canonical correlation for constrained-residuals regression is easily formulated, but the procedure has not been used. The Farrell approach completely generalizes to the case of M outputs and N inputs.

Model Discrimination: For least-squares, several statistics are available to assist in the selection of an appropriate functional form and to guide in choosing which variables to include in the model. For constrained-residuals regression these statistics are not yet developed, although the choice of functional form and of the variables to be included still has to be made. Experience with using least-squares regression on the higher education institutional data indicates that the variation in behavior is so large that most models do very well in terms of R^2 and many of the variables are highly significant. The problem, however, is that there is very little discriminating power in the data between models.

For Farrell's approach, statistics are not yet developed, but the only choice that needs to be made is with respect to the variables to include. The functional form problems do not exist.

Multicollinearity and Heteroscedasticity: The effects of these two problems on regression results are well documented in the literature (D. Farrar and R. Glauber, [1967]; R. Ridker and J. Henning, [1967]). For the Farrell approach, the effects of collinearity between variables in the linear programming model is straightforward. If two constraint variables are highly collinear, the solution to the linear programming problem will be infeasible for certain specifications of the right-hand side of the constraints. For example, it may not be possible to have a high value for the right-hand side of one constraint and a low value for the other constraint. If the objective function variable and one of the constraint variables are highly collinear, no special computational problems arise. With the Farrell method, the effects of collinearity appear explicitly and not implicitly through insignificant regression coefficients. The problem of heteroscedasticity does not arise with Farrell's method, since, as discussed earlier, the magnitude of the variables are appropriately scaled by the objective function variable and the weights (z's). This problem is significant for least-squares, since large observations are given much more weight in determining the regression coefficients, unless the variables are scaled. This scaling, however, causes functional form and interpretation problems.

Sensitivity to Outliers: Of the three methods being discussed, least-squares is the least sensitive to outliers in the data. Both constrained-residuals regression and Farrell's method are forced to the extreme values of the observations. With outliers present in the data, the estimates obtained by either of these two methods are strongly influenced by the extreme points.

However, with Farrell's method of computation it is easy to identify the extreme observations, since all calculations are performed directly on the actual observations and not on a moment-matrix, where the identity of an individual observation is lost as in regression analysis. Therefore, it is easier to check for possible errors in the data. Chapter VI of this paper is devoted to the problem of the sensitivity of results to individual observations.

Least-Cost Modification

The basic computational algorithm used in this study can be modified to find the least-cost method of having a given enrollment with specified institutional characteristics given the observed data. Letting P_i = the unit price of input i, the least-cost algorithm is:

$$\text{Minimize} \quad \sum_{t=1}^{T} z_t \left(\sum_{i=1}^{N} P_i X_{it} \right)$$

$$\text{Subject to:} \quad \sum_{t=1}^{T} z_t Y_{jt} \geq Y_{js} \qquad j = 1,\ldots,M$$

$$\sum_{t=1}^{T} z_t \left(\sum_{i=1}^{N} P_i X_{it} \right) \left(C_{kt} - C_{ks} \right) \leq 0 \qquad k = 1,\ldots,Q_1$$

$$\sum_{t=1}^{T} z_t \left(\sum_{i=1}^{N} P_i X_{it} \right) \left(C_{kt} - C_{ks} \right) \geq 0 \qquad k = Q_1+1,\ldots,Q$$

$$z_t \geq 0 \qquad t = 1,\ldots,T$$

Verbally, the problem is to minimize the total cost of a hypothetical institution constructed as a linear sum of observed institutions, subject to the constraints that the constructed institution has at least as much of every specified enrollment and equals or exceeds the various institutional characteristic constraints.

From the solution values of the z_t's , the cost-minimizing level of each input is given by $X_i^* = \sum_{t=1}^{T} z_t X_{it}$. If certain inputs are considered fixed, they can be included as constraints in the LP model, either as equalities or inequalities if idle capacity is allowed, and enter the objective function only as a fixed constant.

This procedure allows the computation of least-cost methods of producing various enrollment combinations with specified institutional characteristics given factor prices and the production relationships observed from the cross-section of institutions. Instead of minimizing with respect to one input (or maximizing with respect to one output) as done in the basic computational approach, all the inputs are weighted by their unit prices, and their sum is minimized. Alternatively, the quantity $\left(\sum_{i=1}^{N} P_i X_{it}\right)$ in the above formulation can be replaced by the actual total expenditures of the t^{th} institution. This approach also yields information about the cost-minimizing behavior observed for the sample of institutions. These procedures make the appropriate link between the production relationships and the cost relationships for this type of frontier analysis.

V. DESCRIPTIVE RESULTS

The empirical results from this analysis of a wide cross-section of institutions for the fiscal year 1968 are descriptive of the frontier production and cost behavior of the "firms" in the higher education "industry." Prescriptive implications from these results must be made cautiously since no analysis of changes over time for individual institutions has been done. These characteristics of the results are not unique to the computational method being used but rather apply to any type of single-year, cross-sectional analysis of production and cost behavior. In addition, the results are descriptive of the production and cost behavior relative to the observed frontiers, not the true production and cost frontiers. Therefore, the results are conservative estimates of the "efficient" production and cost relationships.

The computations to determine the frontier production relationships do not produce an equation with estimated parameters that relates all of the variables together. Therefore, it is necessary to explicitly evaluate the frontier relationship between any two variables while holding all the other variables constant. This procedure has a disadvantage in that the reporting of results is much more complex and cumbersome than the usual econometric equation form of results. The method used in this study has the advantage, however, that the production and cost relationships are explicitly analyzed in detail.

As a means of presenting the results, the "average" institution within each institutional category is chosen as a base point from which to illustrate the various production and cost relationships. An "average" institution means a hypothetical institution that has values for all of the variables equal to the means of each variable over the particular group of institutions. Analyses are presented in Chapter VII that illustrate the observed differences in production and cost relationships between the average institution and other institutional specifications. The data that describe the average institution for each institutional category are given in Table 3-4 (Chapter III, page 35). The data in this table also illustrate some of the major differences between the types of colleges and universities.

The empirical results are presented and discussed in the following five sections. First, several frontier production relationships are analyzed with respect to the minimum levels of inputs observed for alternative levels and mixes of enrollment. Second, frontier substitution rates between alternative input combinations and frontier transformation rates between different types of enrollment are illustrated. Third, the effects of varying institutional characteristics on the production relationships are analyzed. Fourth, the actual expenditure data of the institutions are utilized to study least-cost behavior with respect to alternative enrollment mixes and institutional characteristics. Fifth, least-cost input combinations are analyzed, and their sensitivity to different relative input prices is illustrated.

Frontier Production Relationships

In order to determine how far the average institution within each institutional category is from the respective frontier production surface, calculations were performed to determine this distance with respect to senior faculty (SFAC), classroom space (CLSP), and full-time undergraduates (UGFT). To illustrate the computational method in more detail, the actual linear programming problem that was solved to determine the minimum number of senior faculty for the average public universities is outlined in Table 5-1. Note that the right-hand side of the constraints are simply the average values for each of the respective variables. The constraints are of the form $\leq$ for the inputs, $\geq$ for the enrollment variables, and either $\leq$ or $\geq$ for the characteristic variables depending on their original production specification.

The results from these calculations are shown in Table 5-2. As an example, the minimum number of senior faculty for a public university with all the characteristics listed in Table 3-4 has the following interpretation. From the data of the observed institutions in the public university sample, it is possible to construct an institution as a weighted sum of the observed universities such that the constructed institution has the same institutional characteristics (or better), the same enrollment levels (or more), and the same input levels (or less) as the average public university. In addition, the hypothetical institution, which lies on the production frontier, has only 651 senior faculty instead of the average 778. The average institution has 20% more senior faculty than the constructed frontier institution with the same characteristics, enrollment, and other input levels. In terms of the student-faculty ratio for this sample of public universities, the average institution has a ratio of 17.7,

TABLE 5-1

Linear Programming Model Example - Public Universities

Minimize $\sum_{t=1}^{37} z_t \, SFAC_t$

Subject to:

$$\sum_t z_t \, JFAC_t \leq 178$$

$$\sum_t z_t \, NPDP_t \leq 276$$

$$\sum_t z_t \, GALB_t \leq 1{,}070$$

$$\sum_t z_t \, CLSP_t \leq 150{,}811$$

$$\sum_t z_t \, LASP_t \leq 387{,}278$$

$$\sum_t z_t \, GASP_t \leq 406{,}179$$

$$\sum_t z_t \, UGPT_t \geq 1{,}041$$

$$\sum_t z_t \, UGFT_t \geq 9{,}539$$

$$\sum_t z_t \, GRAD_t \geq 2{,}697$$

$$\sum_t z_t \, OTHE_t \geq 498$$

$$\left(\sum_t z_t \, SFAC_t \, GOUR_t\right) / \sum_t z_t \, SFAC_t \geq 453$$

$$\left(\sum_t z_t \, SFAC_t \, PSCI_t\right) / \sum_t z_t \, SFAC_t \geq 21$$

$$\left(\sum_t z_t \, SFAC_t \, RETN_t\right) / \sum_t z_t \, SFAC_t \geq 97$$

$$\left(\sum_t z_t \, SFAC_t \, SCLE_t\right) / \sum_t z_t \, SFAC_t \leq 4{,}650$$

$$\left(\sum_t z_t \, SFAC_t \, NFLD_t\right) / \sum_t z_t \, SFAC_t \geq 125$$

TABLE 5-1 (continued)

$$\left\{\sum_t z_t \ SFAC_t \ GRTH_t\right\} / \sum_t z_t \ SFAC_t \leq 7.6$$

$$\left\{\sum_t z_t \ SFAC_t \ PRES_t\right\} \ \sum_t z_t \ SFAC_t \geq 15.0$$

$$\left\{\sum_t z_t \ SFAC_t \ PEXT_t\right\} \ \sum_t z_t \ SFAC_t \geq 5.4$$

$$z_t \geq 0 \quad t = 1, \ldots, 37$$

TABLE 5-2

Frontier Production Relationships by Institutional Type

Variable	Public Universities	Private Universities	Public Comprehensive Colleges	Private Comprehensive Colleges	Public Limited Comprehensive Colleges	Private Highly Selective Liberal Arts Colleges	Private Less Selective Liberal Arts Colleges
Total Enrollment	13,775	7,879	7,829	3,113	2,532	1,173	915
Minimum Senior Faculty	651	452	186	89	98	74	36
Total Enrollment/ Average Senior Faculty	17.7	11.8	21.8	19.8	19.3	12.2	15.0
Total Enrollment/ Minimum Senior Faculty	21.2	17.4	30.2	35.0	25.8	15.9	25.4
Average Senior Faculty/ Minimum Senior Faculty	1.20	1.48	1.39	1.76	1.34	1.30	1.69
Minimum Classroom Space	118,500	61,300	40,000	23,600	19,700	16,300	7,700
Average Classroom Space/ Total Enrollment	10.9	12.0	12.4	24.8	16.5	25.0	24.6
Minimum Classroom Space/ Total Enrollment	8.6	7.8	7.1	7.6	7.8	13.9	8.4
Average Classroom Space/ Minimum Classroom Space	1.27	1.54	1.75	3.29	2.12	1.80	2.92
Maximum Full-time Undergraduates	11,268	5,586	5,873	3,303	2,737	1,436	1,233
Maximum Full-time Undergraduates/ Average Full-time Undergraduates	1.13	1.44	1.35	1.55	1.24	1.29	1.50

while the frontier institution has a student-faculty ratio of 21.2. Similar results are shown in Table 5-2 for the other categories of institutions. The ratio of average senior faculty to minimum senior faculty ranges from 1.20 for the public universities to 1.76 for the private comprehensive colleges. Since total enrollment is the same for both the average and minimum senior faculty institutions, the range of the ratio of average student-faculty ratio to minimum student-faculty ratio is also 1.20 to 1.76.

Results are also given in Table 5-2 for the differences between classroom space at the average institution and at the production frontier. The ratio of classroom space at the average to the frontier, all other variables constant, ranges from 1.27 for the public universities to 3.27 for the private comprehensive colleges. For each institutional type, the percentage decrease from the average to the frontier is greater for the classroom space variable than for the senior faculty variable. It is interesting to note that the dispersion in classroom space per student across institutional types is much smaller on the frontier than for the averages. Also the dispersion for classroom space per student on the frontier is smaller than the dispersion of the student-faculty ratio on the frontier across types of institutions. This implies that classroom space per student is less sensitive than faculty per student to differences in enrollment mixes and institutional characteristics.

From these results, it must be cautioned that inferences such as "public universities are more efficient than private comprehensive colleges since the average is closer to the frontier" <u>cannot</u> be made. This would be possible if the <u>true</u> frontiers were known but each institution is measured relative to an estimated frontier for each institutional type. Therefore, the results simply mean that the production frontier as observed for public universities is much closer to the average of these universities than the frontier as observed for the private comprehensive colleges is to the average private comprehensive college. Results in a later chapter provide some information about cost differences between institutional categories.

Additional information about the production frontier is given in Table 5-2 from a similar analysis done with respect to an enrollment variable, the number of full-time undergraduates. The distance from the frontier institution to the average institution with respect to this variable ranges from a 13% increase in full-time undergraduates for public universities to a 55% increase for private comprehensive colleges.

The results presented in Table 5-2 indicate the distance between the average and frontier production relationships with respect to several key variables. The large distance observed between the average level and the frontier level of senior faculty, for example, implies substantial reductions in cost per student for an institution if it moves towards the production frontier as observed for each sample of institutions.

Since the results given above and the ones to be presented later are based on hypothetical institutions constructed as weighted averages of observed institutions, it is helpful for understanding the computational method and for interpreting the results to illustrate in detail the actual data for the institutions that were selected as components in constructing one of the hypothetical, frontier institutions. Note that all of the institutions in the example are themselves actually on the production frontier but that a weighted combination is needed in order to construct an institution with all of the desired characteristics. The eight observed institutions used to construct the hypothetical institution with the minimum number of senior faculty and with all the other variables equal to the average value for the public comprehensive colleges is given in Table 5-3. For each observed institution the weight (wt.) used in constructing the summation is given along with the values of certain variables and the student-faculty ratio. Only three of the eight institutions have quality ratings (GOUR) less than the average of 366, and three colleges also have a scale (SCLE) less than the average institution (903). The observed student-faculty ratios are all considerably higher than the average (21.8), and two of the observed institutions have student-faculty ratios higher than the ratio for the constructed, frontier institution (30.2). With the data laid out as in Table 5-3, it is easier to get a feel for what the computational algorithm is doing. Since there are observed institutions with higher quality, smaller scale, and larger student-faculty ratios than the constructed institution, the hypothetical institution is not simply a large, low-quality college with a high student-faculty ratio. Rather, the constructed institution, in this example, is an institution with a complex balance of all the variables and with a relatively small number of senior faculty.

The next set of analyses determine the frontier marginal product relationships between changes in the enrollment variables and changes

TABLE 5-3

Illustration of Computational Method - Public Comprehensive Colleges

Institution Number	Weight	Gourman Quality Rating	Institutional Scale	Senior Faculty	Total Enrollment	Total Enrollment / Senior Faculty
45	.02	372	1,809	270	7,085	26.2
64	.02	426	1,692	358	13,036	37.3
67	.19	364	778	244	9,203	37.7
69	.39	365	535	152	4,524	29.8
85	.05	381	1,064	342	9,924	29.0
88	.07	377	1,140	288	8,023	27.8
92	.01	385	1,055	314	9,029	28.8
93	.14	362	816	219	6,583	30.0
"Average" Institution		366	903	258	5,629	21.8
"Minimum Senior Faculty" Institution		366	903	186	5,629	30.2

in the minimum number of senior faculty and classroom space. The initial computations described above gave some information about the distance between the average institution and the frontier institutions, while the computations discussed below provide information about the shape of the production frontier with respect to the enrollment variables, senior faculty, and classroom space. These calculations were done separately for each institutional category, and the results are given in Tables 5-4 through 5-10. The computational procedure is simply to solve the linear programming problem described earlier with alternative values for the right-hand sides of the enrollment constraints. To illustrate the method with the results of the public universities (Table 5-4), the minimum number of senior faculty for an institution with 7,650 full-time undergraduates and all the other variables equal to their means is 595. If full-time undergraduate enrollment is increased to 9,539, the minimum number of senior faculty increases to 651. All of the results in Tables 5-4 through 5-10 have a similar interpretation.

In the discussion of marginal productivities that follows, statements are made which imply that changes in certain variables correspond to changes in other variables. It should be noted that these relationships apply only to the frontier institutions as observed in the samples at one point in time and that this type of empirical analysis cannot indicate whether or not a specific institution can, in fact, exhibit this behavior over time. Several interesting relationships, consistent across institutional categories, emerge from these computations and are discussed below.

(1) For all institutional categories and all the enrollment

TABLE 5-4

Marginal Relationships - Public Universities

Variable	Observation								
	1		2		3		4		5
Full-time Undergraduates (UGFT)	3,820		7,650		9,539		11,480		
Minimum Senior Faculty (SFAC)	594		595		651		788		
ΔUGFT/ΔSFAC		3,830.0		33.8		14.2			
Part-time Undergraduates (UGPT)	835		1,041		1,250		2,080		3,120
Minimum Senior Faculty (SFAC)	651		651		651		651		654
ΔUGPT/ΔSFAC		∞		∞		∞		347.0	
Graduate Students (GRAD)	1,080		2,160		2,697		3,230		
Minimum Senior Faculty (SFAC)	617		618		651		704		
ΔGRAD/ΔSFAC		1,080.0		16.3		10.0			
Specialized Enrollment (OTHE)	0		400		498		600		
Minimum Senior Faculty (SFAC)	602		627		651		684		
ΔOTHE/ΔSFAC		16.0		4.1		3.1			
Full-time Undergraduates (UGFT)	7,650		9,539		11,480				
Minimum Classroom Space (CLSP)	97,060		118,454		151,792				
ΔUGFT/ΔCLSP		.088		.058					

TABLE 5-5

Marginal Relationships - Private Universities

Variable	Observation 1		2		3		4
Full-time Undergraduates (UGFT)	1,630		3,260		4,073		4,890
Minimum Senior Faculty (SFAC)	407		409		452		526
ΔUGFT/ΔSFAC		815.0		18.9		11.0	
Part-time Undergraduates (UGPT)	450		563		675		1,130
Minimum Senior Faculty (SFAC)	452		452		452		453
ΔUGPT/ΔSFAC		∞		∞		455.0	
Graduate Students (GRAD)	2,050		2,565		3,080		4,100
Minimum Senior Faculty (SFAC)	452		452		462		604
ΔGRAD/ΔSFAC		∞		51.5		7.2	
Specialized Enrollment (OTHE)	542		678		814		
Minimum Senior Faculty (SFAC)	419		452		503		
ΔOTHE/ΔSFAC		4.1		2.7			
Full-time Undergraduates (UGFT)	3,260		4,073		4,890		
Minimum Classroom Space (CLSP)	51,000		61,260		73,376		
ΔUGFT/ΔCLSP		.079		.067			

TABLE 5-6

Marginal Relationships - Public Comprehensive Colleges

Variable	Observation								
	1		2		3		4		5
Full-time Undergraduates (UGFT)	3,470		4,335		5,200				
Minimum Senior Faculty (SFAC)	155		186		219				
ΔUGFT/ΔSFAC		27.9		26.2					
Part-time Undergraduates (UGPT)	492		615		738		1,230		1,850
Minimum Senior Faculty (SFAC)	186		186		186		188		197
ΔUGPT/ΔSFAC		∞		∞		296.0		68.9	
Gr[illegible] Students (GRAD)	208		417		521		625		1,040
Minimum Senior Faculty (SFAC)	179		182		186		193		253
ΔGRAD/ΔSFAC		69.7		26.0		15.0		6.9	
Specialized Enrollment (OTHE)	126		158		190		253		316
Minimum Senior Faculty (SFAC)	185		186		187		189		191
ΔOTHE/ΔSFAC		32.0		32.0		31.5		31.5	
Full-time Undergraduates (UGFT)	3,470		4,335		5,200				
Minimum Classroom Space (CLSP)	31,389		39,967		53,462				
ΔUGFT/ΔCLSP		.101		.064					

TABLE 5-7

Marginal Relationships - Private Comprehensive Colleges

Variable	Observation								
	1		2		3		4		5
Full-time Undergraduates (UGFT)	1,710		2,136		2,560				
Minimum Senior Faculty (SFAC)	79		89		99				
ΔUGFT/ΔSFAC		42.6		42.4					
Part-time Undergraduates (UGPT)	368		461		553		922		1,380
Minimum Senior Faculty (SFAC)	89		89		89		95		114
ΔUGPT/ΔSFAC		∞		∞		61.5		24.1	
Graduate Students (GRAD)	159		318		397		477		795
Minimum Senior Faculty (SFAC)	79		83		89		95		120
ΔGRAD/ΔSFAC		39.8		13.3		13.2		12.7	
Specialized Enrollment (OTHE)	C		95		119		143		240
Minimum Senior Faculty (SFAC)	87		87		89		91		101
ΔOTHE/ΔSFAC		∞		12.1		12.0		9.7	
Full-time Undergraduates (UGFT)	1,710		2,136		2,650				
Minimum Classroom Space (CLSP)	19,794		23,596		28,850				
ΔUGFT/ΔCLSP		.112		.081					

TABLE 5-8

Marginal Relationships - Public Limited Comprehensive Colleges

Variable	Observation								
	1		2		3		4		5
Full-time Undergraduates (UGFT)	1,760		2,203		2,640				
Minimum Senior Faculty (SFAC)	82		98		119				
ΔUGFT/ΔSFAC		27.7		20.8					
Part-time Undergraduates (UGPT)	113		141		169		282		423
Minimum Senior Faculty (SFAC)	97		98		99		104		113
ΔUGPT/ΔSFAC		28.0		28.0		22.6		15.7	
Graduate Students (GRAD)	140		175		210				
Minimum Senior Faculty (SFAC)	95		98		103				
ΔGRAD/ΔSFAC		11.7		7.0					
Specialized Enrollment (OTHE)	0		10		13		16		48
Minimum Senior Faculty (SFAC)	97		98.1		98.5		99		105
ΔOTHE/ΔSFAC		9.1		7.5		6.0		5.3	
Full-time Undergraduates (UGFT)	1,760		2,203		2,640				
Minimum Classroom Space (CLSP)	11,935		19,699		35,810				
ΔUGFT/ΔCLSP		.057		.027					

TABLE 5-9

Marginal Relationships - Private Highly Selective Liberal Arts Colleges

Variable	Observation 1		2		3		4		5
Full-time Undergraduates (UGFT)	810		1,114		1,220				
Minimum Senior Faculty (SFAC)	55		74		81				
ΔUGFT/ΔSFAC		16.0		15.2					
Part-time Undergraduates (UGPT)	19		24		29		58		87
Minimum Senior Faculty (SFAC)	74		74		74		74		74
ΔUGPT/ΔSFAC		∞		∞		∞		∞	
Graduate Students (GRAD)	24		30		36		72		108
Minimum Senior Faculty (SFAC)	73		73.4		74		78		83
ΔGRAD/ΔSFAC		15.0		10.0		9.0		7.2	
Specialized Enrollment (OTHE)	0		5		20				
Minimum Senior Faculty (SFAC)	73		74		77				
ΔOTHE/ΔSFAC		5.0		5.0					
Full-time Undergraduates (UGFT)	810		1,114		1,220				
Minimum Classroom Space (CLSP)	12,338		16,328		17,852				
ΔUGFT/ΔCLSP		.076		.070					

TABLE 5-10

Marginal Relationships - Private Less Selective Liberal Arts Colleges

Variable	Observation				
	1		2		3
Full-time Undergraduates (UGFT)	657		822		986
Minimum Senior Faculty (SFAC)	29		36		45
ΔUGFT/ΔSFAC		23.6		16.0	
Part-time Undergraduates (UGPT)	55		69		207
Minimum Senior Faculty (SFAC)	36		36		39
ΔUGPT/ΔSFAC		∞		46.0	
Graduate Students (GRAD)	16		20		80
Minimum Senior Faculty (SFAC)	36		36		40
ΔGRAD/ΔSFAC		∞		15.0	
Specialized Enrollment (OTHE)	0		4		50
Minimum Senior Faculty (SFAC)	36		36		45
ΔOTHE/ΔSFAC		∞		5.1	
Full-time Undergraduates (UGFT)	657		822		986
Minimum Classroom Space (CLSP)	6,030		7,700		10,802
ΔUGFT/ΔCLSP		.0988		.0529	

variables, the marginal productivity relationships ($\Delta UGFT/\Delta SFAC$, $\Delta UGPT/\Delta SFAC,\ldots$) decrease as the minimum number of senior faculty increases. The decreasing marginal product is consistent with economic theory and intuitively sound, since all the other institutional inputs are held constant as the number of senior faculty increases. That is, with all the other inputs the same, an additional senior faculty member at a frontier institution with 180 senior faculty will provide for a greater increase in enrollment than at a frontier institution with 220 senior faculty members. These results imply that increases in senior faculty or any other input will not lead to the same increase in enrollment independent of the levels of all the other inputs. These results illustrate the problems inherent in studies that attempt to determine a single number for the marginal productivity of senior faculty for additional enrollment. The marginal relationships appear to be complex functions of the level and mix of enrollment, the mix of inputs, and (to be shown later) other institutional characteristics.

(2) Frontier institutions with relatively high proportions of graduate enrollment (the public and private universities) appear to increase full-time undergraduate enrollment from low levels to more average levels with few additional senior faculty. For the public universities, UGFT goes from 3,820 to 9,539 with the minimum SFAC increasing by 57, while for the private universities, UGFT increases from 1,630 to 4,073 with 45 additional senior faculty. The remaining institutional categories, which have relatively few graduate students, do not exhibit this type of behavior. For these institutions, the marginal productivity of senior faculty for full-time undergraduates

is more nearly constant for below and above average changes in UGFT. These results imply a strong degree of jointness between graduate students and full-time undergraduates. That is, an additional 1,000 full-time undergraduates means a smaller increase in senior faculty for a frontier institution that has a high proportion of graduate enrollment than for a frontier, primarily undergraduate, institution.

(3) There also appears to be a strong degree of jointness between part-time undergraduates and full-time undergraduates for all institutional categories. For a frontier institution with average (for its institutional type) full-time undergraduate enrollment, part-time undergraduate enrollment can be increased from below average levels to above average levels while the minimum number of senior faculty increases by a very insignificant amount. These variable productivity relationships raise some questions about the use of "full-time equivalent" weighting schemes that usually assume a part-time student equals a constant one-third full-time student regardless of enrollment levels and mixes.

(4) Specialized enrollment, which consists of students enrolled in first-professional, occupational, and extension programs, does not seem to have any joint relationships with any of the other enrollment variables. The productivity of senior faculty with respect to this type of enrollment is comparatively very low for all institutional categories and exhibits varying degrees of declining as the level of specialized enrollment increases.

Also shown in Tables 5-4 through 5-10 are the marginal productivities of classroom space with respect to full-time undergraduate enrollment ($\Delta UGFT/\Delta CLSP$) . The results indicate decreasing marginal productivity rates for all categories of institutions. Note that the variation of the classroom space marginal productivities for the two university categories is much less than the variation of the senior faculty marginal productivities. This implies that the relationship between classroom space and full-time undergraduate enrollment is not affected very much by enrollment mix and enrollment level variations. This relationship is consistent with the earlier observation (see Table 5-2) that classroom space per student is less sensitive than faculty per student to differences i enrollment mixes and levels. The decreasing marginal productivity as classroom space increases is the result of all the other institutional inputs being held at constant levels.

As a means of comparing the marginal productivity rates for senior faculty and classroom space with respect to the number of full-time undergraduates, Table 5-11 gives for each institutional category the frontier marginal product relationships that correspond to a 20% decrease and a 20% increase from the average level of full-time undergraduate enrollment with all the other variables held constant at their means. Also given in the table are the <u>average</u> frontier productivity relationships (defined as the average full-time undergraduate enrollment divided by the corresponding minimum number of senior faculty or minimum square footage of classroom space). For both senior faculty and classroom space and for all institutional types, the marginal productivities are higher relative to the respective average productivities for the private institutions than for the public institutions. The implication is that efficient

TABLE 5-11

Summary of Marginal Relationships by Institutional Type

Institutional Type	Senior Faculty			Classroom Space		
	Average	-20% Marginal	+20% Marginal	Average	-20% Marginal	+20% Marginal
Public Universities	14.7	33.8	14.2	.080	.088	.058
Private Universities	9.0	18.9	11.0	.066	.079	.067
Public Comprehensive Colleges	23.3	27.9	26.2	.108	.101	.064
Private Comprehensive Colleges	24.0	42.6	42.4	.091	.112	.081
Public Limited Comprehensive Colleges	22.5	27.7	20.8	.112	.057	.027
Private Highly Selective Liberal Arts Colleges	15.1	16.0	15.2	.068	.076	.070
Private Less Selective Liberal Arts Colleges	22.8	23.6	16.0	.107	.099	.053

private institutions could increase their average productivity of both senior faculty and classroom space (which are generally lower than those for the public institutions) by increasing the levels of these inputs. In other words, enrollment could be increased significantly with very little additional amounts of these two inputs. The efficient public institutions, on the other hand, would not experience an increase in average productivity from such increases in input levels and in most cases would actually experience a decline in average productivity. More results on these relationships are presented and discussed in Chapter VII.

Frontier Input Substitution and Enrollment Transformation Relationships

Additional information about the shape of the frontier production surface is obtained by analyzing the frontier relationships between alternative mixes of inputs, holding the enrollment variables and the institutional characteristics constant at their means. For illustration, the frontier relationships between senior faculty and classroom space and between senior faculty and junior faculty are shown in Table 5-12. For both relationships and for almost all institutional categories, there does exist some degree of substitution. The two exceptions are with the classroom space-senior faculty mix for private comprehensive colleges and with the junior-senior faculty mix for the public limited comprehensive colleges. For the other institutional types, the substitutions occur at low levels of either junior faculty or classroom space and high levels of senior faculty. Increasing either junior faculty or classroom space beyond a certain point (always less than the average ratio of these inputs to senior

TABLE 5-1.

Input Substitution by Institutional Type

Type	Variable	Observation 1	2	3	4	5
Public Universities	Junior Faculty	100	142	178	214	
	Senior Faculty	698	651	651	651	
	Classroom Space	136,000	143,000	150,811	181,000	
	Senior Faculty	652	651	651	651	
Private Universities	Junior Faculty	80	114	143	172	
	Senior Faculty	535	452	52	452	
	Classroom Space	75,400	94,190	113,000		
	Senior Faculty	484	452	449		
Public Comprehensive Colleges	Junior Faculty	0	5	14	17	20
	Senior Faculty	227	190	186	186	186
	Classroom Space	49,000	52,500	56,000	70,056	84,000
	Senior Faculty	203	196	191	186	186
Private Comprehensive Colleges	Junior Faculty	0	2	6	8	10
	Senior Faculty	128	95	89	89	89
	Classroom Space	46,000	54,000	61,700	77,156	92,500
	Senior Faculty	89	89	89	89	89
Public Limited Comprehensive Colleges	Junior Faculty	1	2	3	4	
	Senior Faculty	98	98	98	98	
	Classroom Space	17,600	20,500	23,400	29,325	35,200
	Senior Faculty	76	74	74	74	74
Private Highly Selective Liberal Arts Colleges	Junior Faculty	0	1	2	3	
	Senior Faculty	75	74	74	74	
	Classroom Space	17,600	20,500	23,400	29,325	35,200
	Senior Faculty	76	74	74	74	74
Private Less Selective Liberal Arts Colleges	Junior Faculty	0	1	8		
	Senior Faculty	38	36	36		
	Classroom Space	13,500	22,500	31,500		
	Senior Faculty	38	36	36		

faculty within each institutional category) does not correspond to a decrease in the minimum number of senior faculty observed in the samples. For public and private universities and comprehensive colleges, the total number of faculty is lower after an initial, low level increase in junior faculty relative to senior faculty with enrollment held constant. The fact that there is observed some degree of substitution between classroom space and senior faculty suggests differences in technical efficiency of classroom utilization between small classes and large classes. An institution with primarily small classes will have a lower ratio of classroom space to senior faculty than an institution with primarily large classes. Which combination is more economically efficient depends on the relative unit prices of senior faculty and classroom space.

The computed frontier relationships between the number of general administration and library personnel (GALB) and the number of senior faculty are similar to the two input substitution relations discussed above. For institutions with extremely low levels of GALB relative to senior faculty, the minimum number of senior faculty is consistently higher than for comparable institutions with GALB at a more average level relative to the number of senior faculty members.

All of the calculated relationships between alternative input mixes imply that input substitution does exist on the frontier production surface as observed from the cross-sectional data. That is, there are alternative, technically efficient input structures observed from the data, and, depending on relative unit input prices, certain of these structures are also allocatively or "cost" efficient. In addition, the results show that for a given institutional struc-
e (a specific set of input, enrollment, and characteristic variables)

the range of substitution between alternative inputs is fairly small. For example, increasing the ratio of junior faculty to senior faculty beyond a 1 to 4 ratio for the universities and a 1 to 14 ratio for the comprehensive colleges does not lead to decreases in the minimum level of senior faculty.

The transformation relationships between alternative types of enrollment can also be studied within the framework of this analysis, and several of these relationships are illustrated in Table 5-13. The frontier transformation rate between part-time undergraduates and full-time undergraduates (ΔUGFT/ΔUGPT) is very small. That is, the level of part-time undergraduates must become fairly large relative to the number of full-time undergraduates before the maximum number of full-time undergraduates decreases significantly for an increase in part-time undergraduates, given constant levels of all institutional inputs and characteristics. This behavior is consistent with the weak productivity relationships discussed earlier for senior faculty with respect to part-time undergraduates <u>given</u> a substantial full-time undergraduate enrollment.

The transformation rates between graduate students and full-time undergraduates (ΔUGFT/ΔGRAD) are somewhat larger than that for part-time and full-time undergraduates, and the rates increase as the ratio of graduates to full-time undergraduates increases. The transformation rates between specialized enrollment and full-time undergraduate enrollment (ΔUGFT/ΔOTHE) are very large and appear to be fairly constant as the ratio of specialized enrollment to full-time undergraduates changes. These results imply as before that there is less "jointness" between specialized and full-time undergraduate

TABLE 5-13

Enrollment Transformations by Institutional Type

Type	Variable	Observation 1	Observation 2	Observation 3
Public Universities	Part-time Undergraduates	835	1,041	2,080
	Full-time Undergraduates	11,268	11,268	11,268
	Graduate Students	2,160	2,697	3,230
	Full-time Undergraduates	11,268	11,268	11,183
	Specialized Enrollment	400	498	600
	Full-time Undergraduates	11,737	11,268	10,774
Private Universities	Part-time Undergraduates	450	563	1,130
	Full-time Undergraduates	5,586	5,586	4,992
	Graduate Students	2,050	2,565	3,080
	Full-time Undergraduates	5,586	5,586	5,586
	Specialized Enrollment	542	678	814
	Full-time Undergraduates	5,668	5,586	5,456
Public Comprehensive Colleges	Part-time Undergraduates	492	615	1,230
	Full-time Undergraduates	5,873	5,873	5,873
	Graduate Students	417	521	625
	Full-time Undergraduates	5,954	5,873	5,778
	Specialized Enrollment	126	158	190
	Full-time Undergraduates	5,905	5,873	5,840

TABLE 5-13 (continued)

Type	Variable	Observation 1	2	3	4
Private Comprehensive Colleges	Part-time Undergraduates / Full-time Undergraduates	368 / 3,303	461 / 3,303	922 / 3,303	
	Graduate Students / Full-time Undergraduates	318 / 3,303	397 / 3,303	477 / 3,302	
	Specialized Enrollment / Full-time Undergraduates	95 / 3,342	119 / 3,303	143 / 3,260	
Public Limited Comprehensive Colleges	Part-time Undergraduates / Full-time Undergraduates	113 / 2,737	141 / 2,737	423 / 2,657	
	Graduate Students / Full-time Undergraduates	140 / 2,778	175 / 2,737	210 / 2,696	
	Specialized Enrollment / Full-time Undergraduates	10 / 2,745	13 / 2,737	16 / 2,727	
Private Highly Selective Liberal Arts Colleges	Part-time Undergraduates / Full-time Undergraduates	19 / 1,436	24 / 1,436	87 / 1,435	
	Graduate Students / Full-time Undergraduates	24 / 1,443	30 / 1,436	108 / 1,313	
	Specialized Enrollment / Full-time Undergraduates	0 / 1,451	5 / 1,436	20 / 1,388	
Private Less Selective Liberal Arts Colleges	Part-time Undergraduates / Full-time Undergraduates	55 / 1,233	69 / 1,233	207 / 1,233	310 / 1,233
	Graduate Students / Full-time Undergraduates	16 / 1,237	20 / 1,233	80 / 1,177	
	Specialized Enrollment / Full-time Undergraduates	0 / 1,234	4 / 1,233	50 / 1,116	

enrollment than between part-time and full-time undergraduate and graduate enrollment. This is consistent with the nature of the programs that the institutions must offer for first-professional, extension, and occupational curricula.

These enrollment transformation results provide an interesting contrast between "joint" and "separable" production processes. Part-time undergraduates, full-time undergraduates, and graduate enrollment are shown to behave as outputs of a joint production process. The transformation rates between these three types of enrollment depend on the levels of all three enrollments. In contrast, full-time undergraduate enrollment and specialized enrollment are shown to behave as outputs of separable production processes. The transformation rate between full-time undergraduates and specialized enrollment does not depend on the levels of these two types of enrollment. The important implication of joint production is that, for example, undergraduate and graduate programs can be offered at one institution for a lower total cost than the sum of costs for one institution offering an undergraduate program only and another institution with only a graduate program.

Institutional Characteristics and Production Relationships

Following the detailed analysis of the frontier relationships between alternative input and enrollment structures with all the institutional characteristic variables held constant, the next step is to study the effect of changes in the characteristic variables on some of the frontier input-enrollment relationships. The hypothesis presented earlier is that institutional quality, scale, program mix, and the other characteristic variables affect the production relationships. The results presented in Table 5-14 support this hypothesis. For all institutional categories (note that these categories are also a type of characteristic variable treated differently from a continuous variable), frontier institutions with higher Gourman quality ratings exhibit a larger number of senior faculty and more square footage of classroom space than frontier institutions with lower quality ratings, holding all the other variables the same. The amount of quality variation that is observed within each institutional category varies, as well as the sensitivity of the minimum number of senior faculty and the minimum level of classroom space to changes in the quality rating.

The frontier relationships between senior faculty and scale (SCLE) are similar across all types of institutions. Increases in scale from low levels up to the average level within each institutional type correspond to fairly substantial decreases in the minimum number of senior faculty. However, increases in scale from the average to higher levels have little effect on the minimum number of senior faculty. This same type of frontier relationship holds consistently for classroom space and scale of the institution. The implication of these empirical results is that productivity gains from increasing the overall size of a frontier institution are experienced only over the range from small to moderate levels of institutional scale. Minimal productivity gains are observed for increases in institutional size from moderate to larger levels.

Some comments at this point concerning the handling of scale in the computational method and the differences between scale and the

TABLE 5-14

Institutional Characteristics and Production Behavior by Institutional Type

Type	Variable	Observation 1	2	3	4	5	6
Public Universities	Gourman Quality Rating	363	453	462	476		
	Senior Faculty	643	651	562	688		
	Institutional Scale	3,720	4,650	5,570	7,450		
	Senior Faculty	671	651	650	650		
	Percent Science Degrees	8.4	12.6	16.8	21.0	25.2	33.6
	Senior Faculty	630	630	635	651	676	781
	Number of Fields	100	125	150			
	Senior Faculty	651	651	680			
	Gourman Quality Rating	363	453				
	Classroom Space	116,854	118,454				
	Institutional Scale	3,720	4,650	5,570			
	Classroom Space	135,912	118,454	117,505			
Private Universities	Gourman Quality Rating	437	546	600			
	Senior Faculty	443	452	484			
	Institutional Scale	4,020	5,034	6,040	8,050		
	Senior Faculty	502	452	437	437		
	Percent Science Degrees	10.0	15.0	20.0	25.0	30.0	10.0
	Senior Faculty	449	449	449	452	464	587
	Number of Fields	72	90	108			
	Senior Faculty	452	452	485			
	Gourman Quality Rating	437	546	600			
	Classroom Space	58,840	61,260	64,942			
	Institutional Scale	4,020	5,034	6,040			
	Classroom Space	69,363	61,260	59,183			

TABLE 5-14 (continued)

Type	Variable	Observation					
		1	2	3	4	5	6
Public Comprehensive Colleges	Gourman Quality Rating	293	366	373	384		
	Senior Faculty	186	186	187	203		
	Institutional Scale	723	903	1,080	1,440		
	Senior Faculty	193	186	186	186		
	Percent Science Degrees	4.8	7.2	9.6	12.0	14.4	19.2
	Senior Faculty	182	182	183	186	193	203
	Number of Fields	34	42	50	58		
	Senior Faculty	186	186	186	196		
	Gourman Quality Rating	293	366	384			
	Classroom Space	39,965	39,967	41,179			
	Institutional Scale	723	903	1,080			
	Classroom Space	40,438	39,967	38,895			
Private Comprehensive Colleges	Gourman Quality Rating	303	379	417	426	436	
	Senior Faculty	88	89	102	109	119	
	Institutional Scale	355	474	592	711	[illegible]	1,180
	Senior Faculty	120	100	89	85	84	84
	Percent Science Degrees	6	12	15	18	24	30
	Senior Faculty	87	87	89	92	100	114
	Number of Fields	24	30	36	42	48	
	Senior Faculty	89	89	92	103	137	
	Gourman Quality Rating	303	379	417			
	Classroom Space	22,867	23,596	26,546			
	Institutional Scale	474	592	711			
	Classroom Space	25,540	23,596	22,337			

TABLE 5-14 (continued)

Type	Variable	Observation 1	2	3	4	5	6
Public Limited Comprehensive Colleges	Gourman Quality Rating	275	344	353	361		
	Senior Faculty	98	98	98	99		
	Institutional Scale	305	349	436	524	610	
	Senior Faculty	115	104	98	96	94	
	Percent Science Degrees	5.2	6.0	7.5	9.0	10.5	
	Senior Faculty	97	97	98	101	104	
	Number of Fields	18	23	28	32		
	Senior Faculty	98	98	99	111		
	Gourman Quality Rating	275	344	361			
	Classroom Space	19,699	19,699	19,759			
	Institutional Scale	349	436	524			
	Classroom Space	23,024	19,699	18,373			
Private Highly Selective Liberal Arts Colleges	Gourman Quality Rating	338	422	464	506		
	Senior Faculty	74	74	78	90		
	Institutional Scale	302	345	431	518	604	
	Senior Faculty	82	77	74	73	73	
	Percent Science Degrees	12.3	15.4	18.5	21.6	24.6	
	Senior Faculty	74	74	74	77	80	
	Number of Fields	17	22	26	31		
	Senior Faculty	74	74	74	80		
	Gourman Quality Rating	338	422	506			
	Classroom Space	16,328	16,328	19,230			
	Institutional Scale	345	431	518			
	Classroom Space	17,484	16,328	16,090			

TABLE 5-14 (continued)

Type	Variable	Observation 1	2	3	4	5	6
Private Less Selective Liberal Arts Colleges	Gourman Quality Rating Senior Faculty	274 36.1	343 36.1	360 36.4	380 43		
	Institutional Scale Senior Faculty	165 40	206 36	248 36			
	Percent Science Degrees Senior Faculty	6 36	10 36	14 37			
	Number of Fields Senior Faculty	13 36	17 36	21 38			
	Gourman Quality Rating Classroom Space	274 7,700	343 7,700	360 7,999	380 10,778		
	Institutional Scale Classroom Space	165 8,742	206 7,700	248 7,498			

marginal productivity relationships (which also imply a change in "size" with respect to one input variable while all the other inputs are held constant) is perhaps necessary. If a scale variable is not explicitly included as one of the characteristic variables, it would not be possible to determine the effects of changes in overall institutional size. Since the hypothetical institutions are constructed as weighted sums of observed institutions, the weights are selected with appropriate magnitude to account for the differences in the overall size of the variables across the institutions. In other words, only the ratios between all of the input and enrollment variables are important in determining the frontier production relationships if the scale variable is excluded. However, with the scale variable treated as a characteristic variable, the magnitude of the institutional scale variable is incorporated into the determination of the frontier relationships. A simple illustration of the treatment of scale is presented in Table 5-15. Solution 3 results from the correct treatment of scale as a characteristic variable while solution 1 shows the problem with treating scale exactly as an input variable. By explicitly including scale as a characteristic variable, it is possible to compute the effect of overall size (or scale) on the production relationships. The marginal productivity relationships between inputs and enrollment variables are independent of the overall size of the institutions and indicate the effect of changing the enrollment mix and/or the ratio between the specific enrollment variable and one of the inputs.

The frontier relationships between the percent science degrees of total degrees granted (PSCI) and the number of senior faculty

TABLE 5-15

Illustration of Institutional Scale
Treated as a Characteristic Variable

Variable	Institution A	Institution B	Institution C
Senior Faculty (SFAC)	10	70	100
Full-time Undergraduates (UGFT)	200	1,562	2,500
Institutional Scale (SCLE)	1,000	5,000	10,000
UGFT/SFAC	20.0	22.3	25.0

Problem: Minimize SFAC with UGFT $\geq$ 1,250 and SCLE $\leq$ 5,000

Variable	Solution 1	Solution 2	Solution 3
Senior Faculty	50	50	56
Full-time Undergraduates	1,250	1,250	1,250
Institutional Scale	5,000	*10,000*	5,000
Institution A wt.	0.0	0.0	0.0
Institution B wt.	0.0	0.0	0.8
Institution C wt.	0.5	0.5	0.0

Solution 1 results from treating SCLE as a <u>regular input</u> variable.

Solution 2 illustrates the violation of the SCLE constraint resulting from solution 1 with SCLE treated as a characteristic variable.

Solution 3 results from treating SCLE as a <u>characteristic</u> variable.

are also similar across institutional types. At low levels of PSCI, increases in the percentage of science degrees correspond to very small or zero increases in the minimum number of senior faculty. However, as the magnitude of PSCI increases, the sensitivity of the minimum level of senior faculty to changes in the percentage of science degrees increases greatly.

The frontier relationships between the number of fields granting degrees (NFLD) and the number of senior faculty are almost identical to the above results across the institutional types. Up to the average number of fields, increases in the number of fields correspond to small or zero increases in the minimum number of senior faculty. Beyond the average number of fields, additional fields require a substantially larger minimum number of senior faculty.

The effects of the research commitment variable (PRES) and the public service involvement variable (PEXT) are fairly significant for the public and private universities, as shown in Table 5-16. Due to the relatively small amount of research and public service involvement for the other categories of four-year colleges, there is little significant change in the minimum number of senior faculty corresponding to changes in these two variables for these institutions. For the universities, there appears to be a jointness effect between research and enrollment, since the increase in the minimum number of senior faculty is much smaller for changes in the research percentage at low levels of PRES than at higher levels of PRES. To a lesser degree, there is also a jointness effect between public service involvement and the level of enrollment.

The results for both the enrollment growth (GRTH) and enrollment retention (RETN) variables also exhibit some interesting tendencies

TABLE 5-16

University Research, Public Service, and Production Behavior

Type	Variable	Observation 1	2	3	4	5
Public Universities	Percent Research Revenues	9	12	15	18	21
	Senior Faculty	647	648	651	657	696
	Percent Public Service Expenditures	3.0	4.3	5.4	6.5	8.0
	Senior Faculty	639	645	651	657	665
Private Universities	Percent Research Revenues	18	23	28	34	
	Senior Faculty	437	437	452	531	
	Percent Public Service Expenditures	1.0	1.2	1.5	1.8	2.0
	Senior Faculty	452	452	452	452	452

and are illustrated in Table 5-17. Although the relationships are not significant for the public universities and the private comprehensive colleges, frontier institutions with above average retention rates exhibit a larger number of senior faculty than frontier institutions with average retention rates. Frontier institutions with below average retention rates, however, do not seem to have fewer senior faculty. All the institutional categories except public universities and public comprehensive colleges exhibit a larger minimum number of senior faculty for institutions with a below average rate of enrollment growth than for institutions with average growth rates of enrollment. Frontier average-growth institutions have fewer senior faculty than slower growing, frontier institutions, with all the other variables the same. For frontier institutions with above average enrollment growth rates, the number of senior faculty is about the same as for frontier institutions with average enrollment growth.

The effects of changes in the Gourman quality rating and of changes in the scale of the institution on the minimum square footage of classroom space are also illustrated in Table 5-14. The relationships are very similar to the effects of quality and scale on the minimum number of senior faculty discussed earlier, and the relationships are consistent across all the institutional categories. Except for the private comprehensive colleges, scale has a greater effect on the minimum level of classroom space than on the minimum number of senior faculty.

The results of this section show that all of the characteristic variables (institutional quality, percent science degrees, enrollment

TABLE 5-17

Enrollment Growth, Enrollment Retention, and Production Behavior by Institution Type

Type	Variable	Observation 1	2	3	4	5	6	7
Public Universities	Enrollment Growth Rate	3.0	4.5	6.0	9.0	15.0		
	Senior Faculty	710	663	651	651	651		
	Enrollment Retention	60	77	97	116	130		
	Senior Faculty	651	651	651	651	651		
Private Universities	Enrollment Growth Rate	1.0	1.5	2.0	2.5	3.1	3.9	4.7
	Senior Faculty	605	523	490	470	461	452	452
	Enrollment Retention		78	98	118			
	Senior Faculty		452	452	625			
Public Comprehensive Colleges	Enrollment Growth Rate	1.0	4.0	8.0	12.0			
	Senior Faculty	231	202	186	186			
	Enrollment Retention		64	80	96			
	Senior Faculty		186	186	188			
Private Comprehensive Colleges	Enrollment Growth Rate	0.0	1.0	2.4	3.0	3.6		
	Senior Faculty	160	95	90	89	88		
	Enrollment Retention		68	85	102	110		
	Senior Faculty		89	89	89	89		
Public Limited Comprehensive Colleges	Enrollment Growth Rate	3.0	6.0	8.5	10.6	12.7		
	Senior Faculty	117	102	99	98	98		
	Enrollment Retention		54	68	82	90	100	
	Senior Faculty		97	98	101	104	117	

TABLE 5-17 (continued)

Type	Variable	Observation						
		1	2	3	4	5	6	7
Private Highly Selective Liberal Arts Colleges	Enrollment Growth Rate	1.0	2.0	2.5	3.0	6.0		
	Senior Faculty	82	76	74	72	70		
	Enrollment Retention		61	76	91	95		
	Senior Faculty		74	74	83	85		
Private Less Selective Liberal Arts Colleges	Enrollment Growth Rate		2.3	2.9	3.5			
	Senior Faculty		37	36	35			
	Enrollment Retention		57	71	85			
	Senior Faculty		36	36	38			

retention, institutional scale, number of programs, enrollment growth, research commitment, and public service involvement) do significantly influence the frontier production relationships. This observed behavior has strong implications for the comparison of costs across institutions. Characteristic variables should be included in any comparison of costs in order to avoid labeling a high cost institution inefficient if the high costs are the result of a particular set of characteristics variables (i.e., high quality, a large number of science programs,...).

Least-Cost Relationships

The link between the production analysis described in detail above and a cost analysis can be made in two ways given the data available for this cross-section of institutions. One method is to use per unit price data and calculate the resulting cost from a given set of inputs. Another procedure is to use the actual expenditure data reported by the institutions. Both methods are used in this study since each procedure allows a slightly different aspect of cost behavior to be analyzed. The use of input prices does not capture all of the institution's costs, since not all the inputs are included in the model, and the per unit prices are only approximated for the building space variables, especially. The behavior of <u>total</u> expenditures can be analyzed, therefore, by using the actual expenditure data for each institution. However, use of the unit price approach makes possible an analysis of the effect of different relative price ratios on the least-cost input mix. The remainder of this section is a description of the behavior of total expenditures for all the categories of institutions. The following section describes the results of the analysis of input price ratios and least-cost input mixes.

Using the actual educational and general expenditures of the institutions, the average cost and least-cost per student (on a total enrollment basis) are determined for the institutions with average enrollment and average characteristics in each institutional category. These expenditure figures include the cost of instruction and departmental research, extension and public service, libraries, physical plant maintenance and operation, general administration, general institutional expense and student services, organized activities relating to educational departments, organized research, and other sponsored programs. The results are given in Table 5-18. The minimum-cost institutions have expenditures that range from slightly less than one-half the expenditures of the average-cost institutions (private universities) to about two-thirds the average-cost institution's expenditures (limited comprehensive colleges).

To further illustrate the least-cost behavior of the samples of institutions, Table 5-19 shows observed minimum costs per student (average cost) and minimum costs of additional students (marginal cost) for changes in both full-time undergraduate and graduate enrollment. These results are generated by first calculating the minimum cost per student with all enrollment and characteristic variables at their means. The second step is to calculate the minimum cost per student if full-time undergraduate enrollment is 20% less than the average and similarly if full-time undergraduate enrollment is 20% higher, graduate enrollment 20% lower, and graduate enrollment 20% higher than the average enrollment levels. As the results indicate, the

TABLE 5-18

Minimum Expenditures by Institutional Type

Variable	UNIV* PUB	UNIV PRI	COMP PUB	COMP PRI	LIMC PUB	LIBA PRI	OLBA PRI
Total Cost/Total Enrollment	$2,590	$5,350	$1,120	$1,320	$1,120	$2,380	$1,440
Minimum Cost/Total Enrollment	1,760	2,420	670	782	787	1,540	842
Total Cost/Minimum Cost	1.47	2.20	1.67	1.69	1.42	1.55	1.72

*Key to headings:

UNIV-PUB = Public Universities
UNIV-PRI = Private Universities
COMP-PUB = Public Comprehensive Colleges
COMP-PRI = Private Comprehensive Colleges

LIMC-PUB = Public Limited Comprehensive Colleges
LIBA-PRI = Private Highly Selective Liberal Arts Colleges
OLBA-PRI = Private Less Selective Liberal Arts Colleges

TABLE 5-19

Average and Marginal Costs by Institutional Type

Type	Variable	Observation 1		2		3
Public Universities	Full-time Undergraduates	7,650		9,539		11,480
	Average Cost	$1,960		$1,760		$1,640
	Marginal Cost		$ 498		$ 840	
	Graduate Students	2,160		2,697		3,230
	Average Cost	1,690		1,760		1,890
	Marginal Cost		3,350		5,380	
Private Universities	Full-time Undergraduates	3,260		4,073		4,890
	Average Cost	2,700		2,420		2,200
	Marginal Cost		4		5	
	Graduate Students	2,050		2,565		3,080
	Average Cost	2,470		2,420		2,440
	Marginal Cost		1,800		2,580	
Public Comprehensive Colleges	Full-time Undergraduates	3,470		4,335		5,200
	Average Cost	700		670		680
	Marginal Cost		497		730	
	Graduate Students	417		521		625
	Average Cost	665		670		706
	Marginal Cost		960		2,690	
Private Comprehensive Colleges	Full-time Undergraduates	1,710		2,136		2,560
	Average Cost	775		782		806
	Marginal Cost		845		970	
	Graduate Students	318		397		477
	Average Cost	785		782		800
	Marginal Cost		760		1,380	

TABLE 5-19 (continued)

Type	Variable	Observation				
		1		2		3
Public Limited Comprehensive Colleges	Full-time Undergraduates	1,760		2,203		2,640
	Average Cost	805		787		775
	Marginal Cost		700		705	
	Graduate Students	140		175		210
	Average Cost	765		787		810
	Marginal Cost		2,280		2,570	
Private Highly Selective Liberal Arts Colleges	Full-time Undergraduates	810		1,114		1,220
	Average Cost	1,530		1,540		1,545
	Marginal Cost		1,545		1,600	
	Graduate Students	24		30		108
	Average Cost	1,535		1,540		1,540
	Marginal Cost		1,670		1,540	
Private Less Selective Liberal Arts Colleges	Full-time Undergraduates	658		822		986
	Average Cost	825		842		845
	Marginal Cost		915		921	
	Graduate Students	16		20		24
	Average Cost	842		842		842
	Marginal Cost		842		842	

marginal cost of an additional student (either a full-time undergraduate or a graduate student) at a frontier institution is lower if the change is from below the average level of that enrollment variable towards the average than if the change is from the average to an above-average enrollment level. With respect to some of the input production relationships, this behavior is consistent with the decreasing marginal productivity rates discussed earlier.

As shown in Table 5-20, the relationships between several of the institutional characteristic variables and the minimum cost per student (on a total enrollment basis) are similar to the production relationships between senior faculty, classroom space, and the same characteristic variables. Frontier institutions with a high quality rating, small scale, or heavily science-oriented programs are shown to have a significantly higher cost per student than other institutions.

The important implication of these least-cost results is that the frontier average costs per student and the marginal costs per student are not constant for each sample of institutions. Both average and marginal costs are shown to depend on enrollment levels and mixes and institutional characteristic variable levels. These cost relationships indicate the reason for the wide range of estimates that result from studies that attempt to determine the average cost per student at one or several institutions. Also, these results suggest that extreme caution should be exercised in comparing costs across institutions so that all the enrollment mix and level and institutional characteristic differences are taken into account.

TABLE 5-20

Institutional Characteristics and Average Costs by Institutional Type

Type	Variable	Observation 1	Observation 2	Observation 3	Observation 4
Public Universities	Gourman Quality Rating	363	453	462	476
	Average Cost	$1,740	$1,760	$1,800	$1,970
	Institutional Scale	3,720	4,650	5,570	
	Average Cost	1,875	1,760	1,755	
	Percent Science Degrees	13.0	21.0	34.0	
	Average Cost	1,755	1,760	2,020	
Private Universities	Gourman Quality Rating	437	546	600	615
	Average Cost	2,320	2,420	2,890	3,100
	Institutional Scale	4,020	5,034	6,040	
	Average Cost	2,620	2,420	2,320	
	Percent Science Degrees	20.0	25.0	40.0	
	Average Cost	2,380	2,420	3,160	
Public Comprehensive Colleges	Gourman Quality Rating	346	366	373	384
	Average Cost	670	670	695	780
	Institutional Scale	723	903	1,080	
	Average Cost	722	670	670	
	Percent Science Degrees	9.6	12.0	19.2	
	Average Cost	650	670	797	
Private Comprehensive Colleges	Gourman Quality Rating	303	379	417	436
	Average Cost	774	782	902	996
	Institutional Scale	474	592	711	
	Average Cost	788	782	782	
	Percent Science Degrees	9.0	15.0	30.0	
	Average Cost	770	782	932	

TABLE 5-20 (continued)

Type	Variable	Observation 1	2	3	4
Public Limited Comprehensive Colleges	Gourman Quality Rating	275	344	353	361
	Average Cost	787	787	790	794
	Institutional Scale	305	436	610	
	Average Cost	954	787	707	
	Percent Science Degrees	5.2	8.0	10.5	
	Average Cost	770	787	787	
Private Highly Selective Liberal Arts Colleges	Gourman Quality Rating	338	422	464	506
	Average Cost	1,450	1,540	1,700	2,010
	Institutional Scale	302	431	604	
	Average Cost	1,580	1,540	1,530	
	Percent Science Degrees	12.3	15.0	24.6	
	Average Cost	1,490	1,540	1,810	
Private Less Selective Liberal Arts Colleges	Gourman Quality Rating	274	343	360	380
	Average Cost	826	842	898	1,040
	Institutional Scale	165	206	248	
	Average Cost	851	842	841	
	Percent Science Degrees	6	10	14	
	Average Cost	834	842	855	

Least-Cost Input Combinations

Using the actual expenditure data described in the previous section to calculate the least-cost results, the input variables were not explicitly included in the analysis. However, it is possible to compute the input levels for the constructed, least-cost institutions from the solution of the linear programming problem. Table 5-21 shows the least-cost institutional input structures and the input levels of the average institutions for each institutional category. For each input, the ratio between the level of that input at the average institution and at the least-cost institution is also given. The wide variation in these ratios for each institutional category implies that the observed least-cost institutions' input levels are not simply a neutral transformation of the average institutions' input structures. The mix of inputs, as well as the level of inputs, is considerably different between the average and least-cost institutions, although they have identical enrollment and characteristic variables. By comparing the results given in Table 5-2 to those in Table 5-21, it is interesting to note that the least-cost institutions have significantly more senior faculty than the constructed institutions with the minimum number of senior faculty for all institutional categories except the public universities. Although senior faculty is the most expensive input (it accounts for the largest proportion of total costs), the least-cost institutions are not simply institutions with the minimum level of senior faculty.

The next step in the analysis of least-cost input combinations is to determine the sensitivity of the least-cost input mix to changes

TABLE 5-21

Least-Cost Input Combinations by Institutional Type

Type	Institution	SFAC*	JFAC	NPDP	GALB	CLSP	LASP	GASP
Public Universities	Least-Cost	648	149	291	635	139,800	334,000	321,400
	Average	778	178	276	1,070	150,800	387,300	406,200
	Ratio: Average/ Least-Cost	1.20	1.19	0.94	1.68	1.07	1.15	1.26
Private Universities	Least-Cost	508	82	207	599	127,300	190,900	271,100
	Average	670	143	180	1,209	94,200	211,800	298,700
	Ratio: Average/ Least-Cost	1.31	1.74	0.86	2.01	0.74	1.11	1.10
Public Comprehensive Colleges	Least-Cost	191	3	11	41	58,100	58,400	59,600
	Average	258	17	47	177	70,100	87,300	101,500
	Ratio: Average/ Least-Cost	1.35	5.66	4.27	4.31	1.21	1.49	1.70
Private Comprehensive Colleges	Least-Cost	115	2	9	101	55,000	43,100	68,500
	Average	157	8	25	146	77,200	47,500	87,800
	Ratio: Average/ Least-Cost	1.36	4.00	2.77	1.44	1.40	1.10	1.28
Public Limited Comprehensive Colleges	Least-Cost	104	1	39	90	34,000	50,400	46,400
	Average	131	3	24	97	41,700	40,300	53,800
	Ratio: Average/ Least-Cost	1.26	3.00	0.62	1.08	1.23	0.80	1.16

TABLE 5-21 (continued)

Type	Institution	SFAC*	JFAC	NPDP	GALB	CLSP	LASP	GASP
Private Highly Selective Liberal Arts Colleges	Least-Cost	86	1	3	36	20,600	21,200	40,000
	Average	96	2	10	122	29,300	33,400	56,600
	Ratio: Average/ Least-Cost	1.11	2.00	3.33	3.38	1.42	1.58	1.42
Private Less Selective Liberal Arts Colleges	Least-Cost	50	0	3	35	15,700	11,500	15,600
	Average	61	1	8	56	22,500	18,300	31,400
	Ratio: Average/ Least-Cost	1.22	∞	2.67	1.60	1.43	1.59	2.01

*Key to headings:

SFAC = Senior Faculty

JFAC = Junior Faculty

NPDP = Nonprofessional Departmental Personnel

GALB = General Administration Personnel

CLSP = Classroom Space

LASP = Laboratory Space

GASP = Administration Office Space

in relative input prices. Since the analysis of the production relationships discussed earlier indicated varying degrees of input substitution, one would expect relative input prices to have an effect on least-cost input structures. Using alternative sets of input unit prices and the actual input levels, a cost figure is computed for each institution, and then the same procedure is used as before to determine the least-cost input structure for each institutional type. The alternative sets of input unit prices and the resulting least-cost input levels are given in Table 5-22. The least-cost input structure given in Table 5-21 is also shown in Table 5-22 for comparison with the other input structures. Increasing the unit prices of non-professional departmental (NPDP) and general administration personnel (GALB) relative to the other input prices decreases the resulting least-cost number of NPDP and GALB and increases the number of senior faculty for all institutional categories except the public and private comprehensive colleges. Doubling the unit prices of all three building space variables (CLSP, LASP, and GASP) relative to the other input prices has the expected result of decreasing, to varying degrees, the least-cost number of square footage of building space and, in some cases, of increasing the number of senior faculty. The exceptions are the public comprehensive colleges and the highly selective liberal arts colleges. For all the categories of institutions except the private universities and the private comprehensive colleges, the first set of input prices yields a least-cost input structure very similar to the least-cost input levels resulting from the analysis of the actual expenditure data.

TABLE 5-22

Input Prices and Input Combinations by Institutional Type

Variable	Least-Cost Input Level	Input		Input		Input	
		Price	Level	Price	Level	Price	Level
PUBLIC UNIVERSITIES:							
Senior Faculty	648	15,000	667	15,000	768	10,800	784
Junior Faculty	149	5,600	120	5,600	122	5,600	122
Nonprofessional Departmental Personnel	291	4,000	268	7,200	224	7,200	213
General Admin. Personnel	635	4,000	622	8,000	476	8,000	451
Classroom Space	139,800	3.5	134,500	3.5	136,700	7.0	137,500
Laboratory Space	334,000	4.5	357,700	4.5	272,200	9.0	274,700
Admin. Office Space	321,400	4.0	331,100	4.0	328,200	8.0	323,400
PRIVATE UNIVERSITIES:							
Senior Faculty	508	15,000	462	15,000	476	15,000	466
Junior Faculty	82	5,600	107	5,600	107	5,600	106
Nonprofessional Departmental Personnel	207	4,000	114	7,000	99	4,000	106
General Admin. Personnel	599	4,000	624	7,000	589	4,000	625
Classroom Space	127,300	3.5	76,200	3.5	78,000	7.0	77,200
Laboratory Space	190,900	4.5	126,100	4.5	124,100	9.0	117,100
Admin. Office Space	271,100	4.0	214,600	4.0	214,800	8.0	210,000
PUBLIC COMPREHENSIVE COLLEGES:							
Senior Faculty	191	13,000	189	13,000	188	13,000	189
Junior Faculty	3	5,000	2	5,000	2	5,000	2
Nonprofessional Departmental Personnel	11	4,000	12	7,000	12	4,000	12

TABLE 5-22 (continued)

Variable	Least-Cost Input Level	Input		Input		Input	
		Price	Level	Price	Level	Price	Level
General Admin. Personnel	41	4,000	43	4,000	41	4,000	43
Classroom Space	58,100	3.5	57,100	3.5	57,200	7.0	57,100
Laboratory Space	58,400	4.5	57,200	4.5	56,900	9.0	57,300
Admin. Office Space	59,600	4.0	59,100	4.0	58,700	8.0	59,200
PRIVATE COMPREHENSIVE COLLEGES:							
Senior Faculty	115	13,000	89	13,000	88	13,000	88
Junior Faculty	2	5,000	3	5,000	3	5,000	3
Nonprofessional Departmental Personnel	9	4,000	21	7,000	21	4,000	18
General Admin. Personnel	101	4,000	26	7,000	26	4,000	44
Classroom Space	55,000	3.5	48,900	3.5	48,700	7.0	45,600
Laboratory Space	43,100	4.5	39,900	4.5	39,200	9.0	35,800
Admin. Office Space	68,500	4.0	56,300	4.0	56,300	8.0	54,500
PUBLIC LIMITED COMPREHENSIVE COLLEGES:							
Senior Faculty	104	13,000	101	13,000	111	13,000	109
Junior Faculty	1	5,000	1	5,000	1	5,000	1
Nonprofessional Departmental Personnel	39	4,000	24	7,000	19	4,000	21
General Admin. Personnel	90	4,000	66	7,000	50	4,000	54
Classroom Space	34,000	3.5	36,400	3.5	34,900	7.0	34,700
Laboratory Space	50,400	4.5	33,800	4.5	28,600	9.0	27,500
Admin. Office Space	46,400	4.0	45,300	4.0	49,300	8.0	46,600

TABLE 5-22 (continued)

Variable	Least-Cost Input Level	Input		Input		Input	
		Price	Level	Price	Level	Price	Level
PRIVATE HIGHLY SELECTIVE LIBERAL ARTS COLLEGES:							
Senior Faculty	86	14,000	85	14,000	87	14,000	86
Junior Faculty	1	5,000	1	5,000	1	5,000	1
Nonprofessional Departmental Personnel	3	4,000	5	7,000	5	4,000	5
General Admin. Personnel	36	4,000	33	7,000	21	4,000	28
Classroom Space	20,600	3.5	20,900	3.5	21,200	7.0	21,700
Laboratory Space	21,200	4.5	21,300	4.5	22,800	9.0	23,200
Admin. Office Space	40,000	4.0	36,700	4.0	36,400	8.0	33,600
PRIVATE LESS SELECTIVE LIBERAL ARTS COLLECES:							
Senior Faculty	50	13,000	42	13,000	42	13,000	43
Junior Faculty	0	5,000	1	5,000	1	5,000	1
Nonprofessional Departmental Personnel	3	4,000	3	7,000	2	4,000	3
General Admin. Personnel	35	4,000	20	7,000	17	4,000	29
Classroom Space	15,700	3.5	13,400	3.5	13,700	7.0	13,000
Laboratory Space	11,500	4.5	9,500	4.5	11,000	9.0	7,600
Admin. Office Space	15,600	4.0	19,300	4.0	21,700	8.0	14,100

The results in this section illustrate quite clearly that relative input prices do have an effect on the least-cost input structures of higher education institutions. These results imply that recommendations to simply lower the number of senior faculty in order to decrease costs may not have the effect of moving the institutions toward the cost frontier. It is necessary to consider all of the institutional inputs, their relative prices, and the production behavior of the institutions in order to determine appropriate least-cost input structures for a desired set of enrollment and institutional characteristics. Also, since the marginal productivity relationships discussed earlier in this chapter were shown to depend on enrollment mixes and institutional characteristics, least-cost input structures may vary for alternative enrollment mixes and institutional characteristic specifications. These results imply that cost-reducing strategies may not be universally applicable; for example, cost reduction may require a different input structure for a high-quality institution than for a low-quality institution.

In order to construct a measure of allocative efficiency for each institution, it is necessary to have data on the actual per unit costs of all the inputs for every institution. Although salary data exist on the HEGIS file for faculty, no data are currently available on other professional and all non-professional personnel or on the building space inputs. With this lack of data, no attempt is made to compute measures of allocative efficiency for the institutions in the samples.

A Graphic Example and a Summary of the Results

To further illustrate the generated frontier production relationships for the public comprehensive colleges, Figures 5-1 through 5-7 are presented on the following pages. To generate the productivity curves shown in Figure 5-1, the senior faculty variable is placed in the linear programming objective function and the right-hand side of the enrollment constraints are varied. The point on the graph labelled "AVE" represents the minimum number of senior faculty observed for a public comprehensive college with all other variables equal to their means. The curve labelled "UGPT" represents the frontier production relationship between increases in the number of part-time undergraduates and the minimum number of senior faculty with all other variables held constant at mean values. The other curves labelled "UGFT," "GRAD," and "OTHE" have similar interpretations for the respective enrollment variables. The vertical axis is total enrollment in order to illustrate the differences in the productivity curves resulting from changes in each of the four enrollment variables.

Figures 5-2 through 5-7 illustrate several other frontier production relationships. These graphs indicate the form of the production relationships generated by the Farrell method. Although the "shapes" of the relationships between alternative pairs of variables are varied, the generated productivity curves (Figures 5-2 and 5-3), isoquant curves (Figures 5-4 and 5-5), and transformation curves (Figures 5-6 and 5-7) behave as suggested by the microeconomic theory of the firm. Also the characteristic variables affect the production relationships as hypothesized although the shifts are not independent of the levels of inputs and outputs.

FIGURE 5-1

Faculty-Enrollment Productivity Curves
by Type of Enrollment for Public Comprehensive Colleges

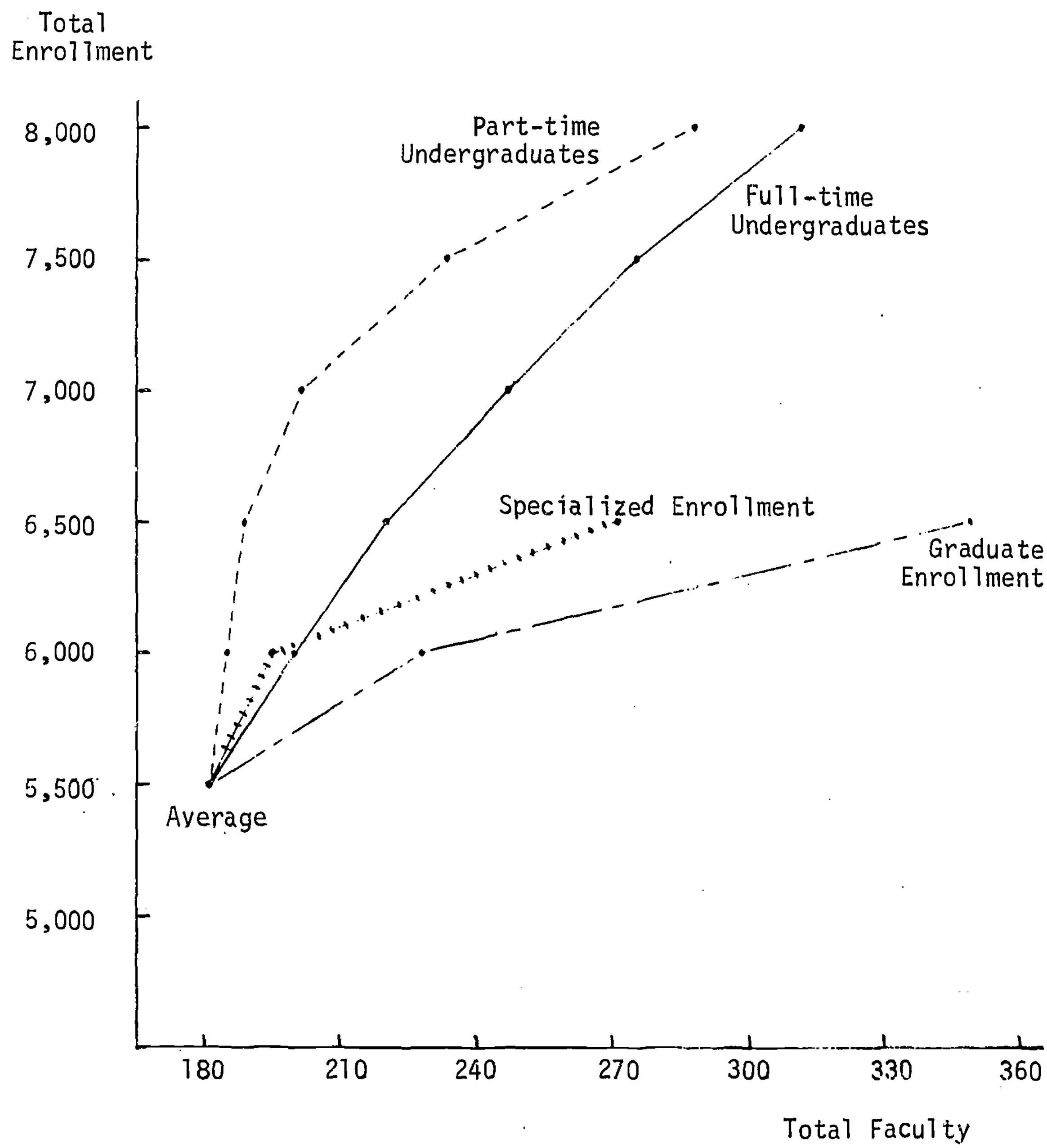


FIGURE 5-2

Faculty - Full-time Undergraduate Enrollment Productivity Curves by Institutional Quality for Public Comprehensive Colleges

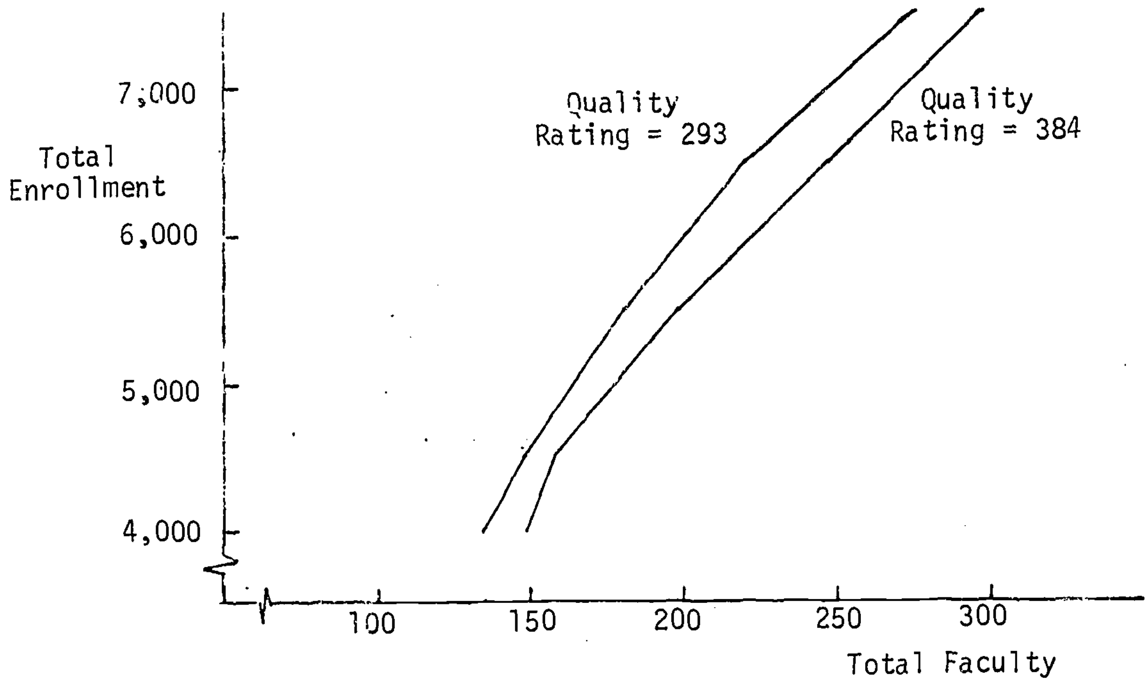


FIGURE 5-3

Classroom Space - Full-time Undergraduate Enrollment Productivity Curves by Institutional Size for Public Comprehensive Colleges

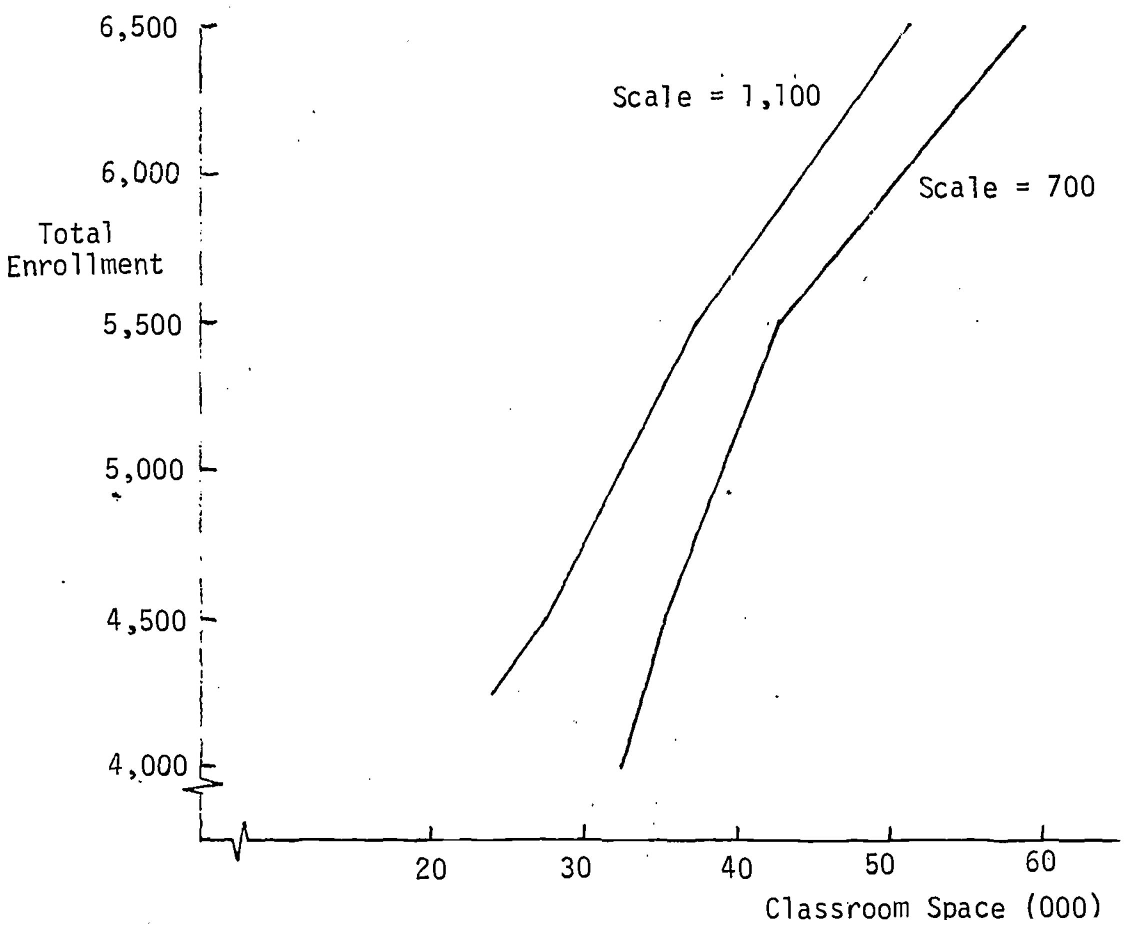


FIGURE 5-4

Junior Faculty - Senior Faculty Isoquant Curves by Percent Science Degrees of Total Degrees Granted for Public Comprehensive Colleges

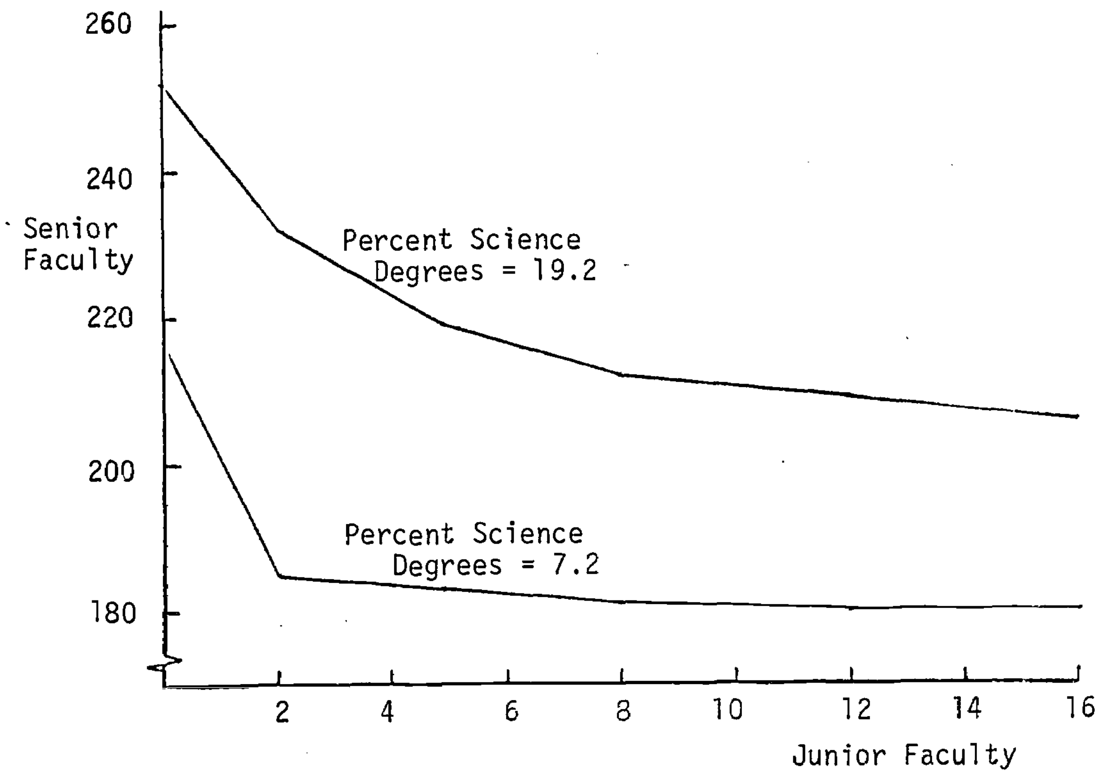


FIGURE 5-5

Classroom Space - Senior Faculty Isoquant Curves by Number of Fields Granting Degrees for Public Comprehensive Colleges

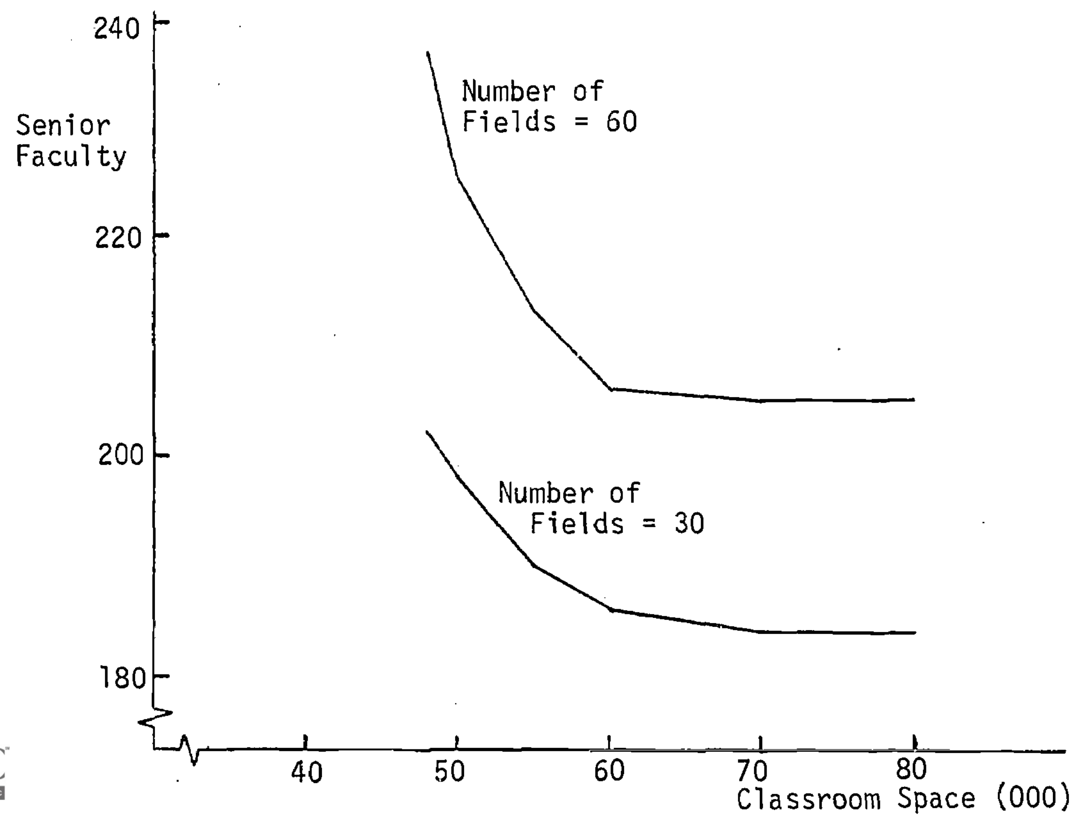


FIGURE 5-6

Specialized Enrollment - Full-time Undergraduate Transformation Curves by Percent Research Revenues of Total Revenues for Public Comprehensive Colleges

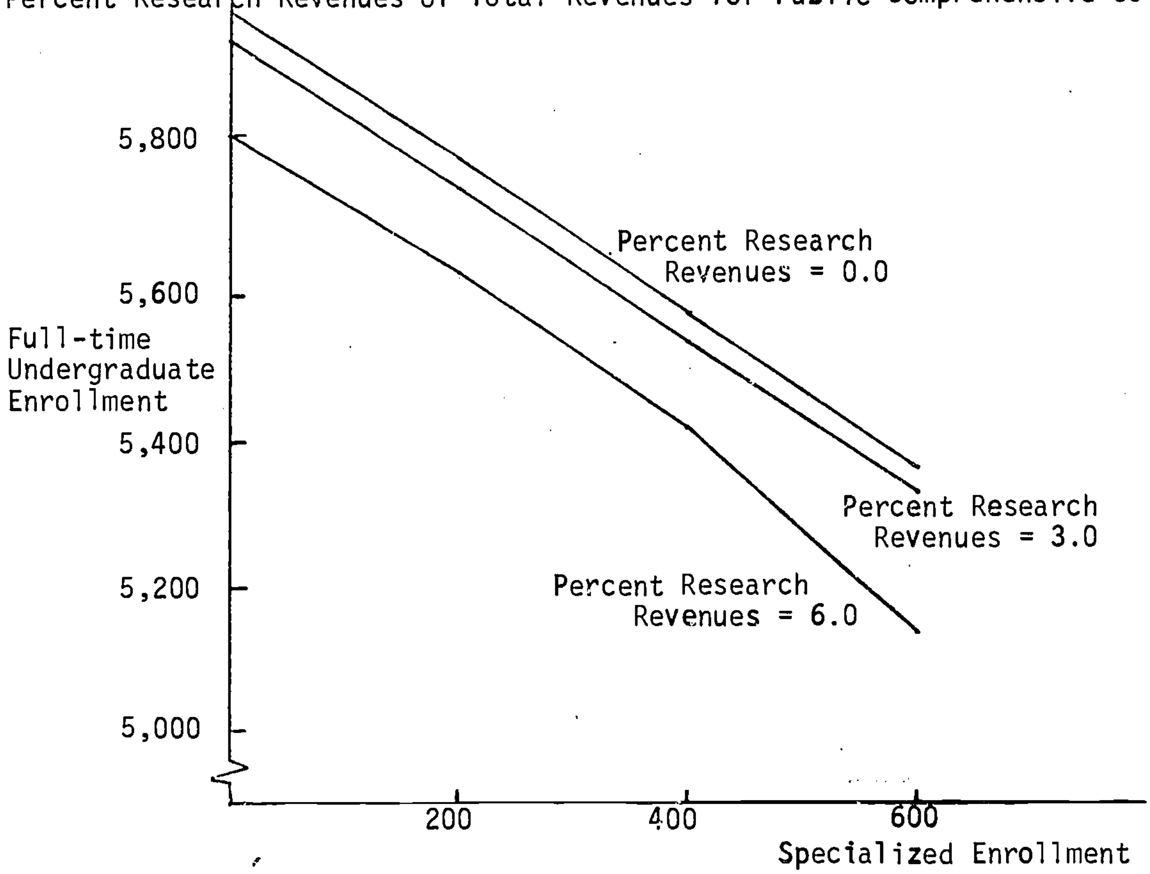
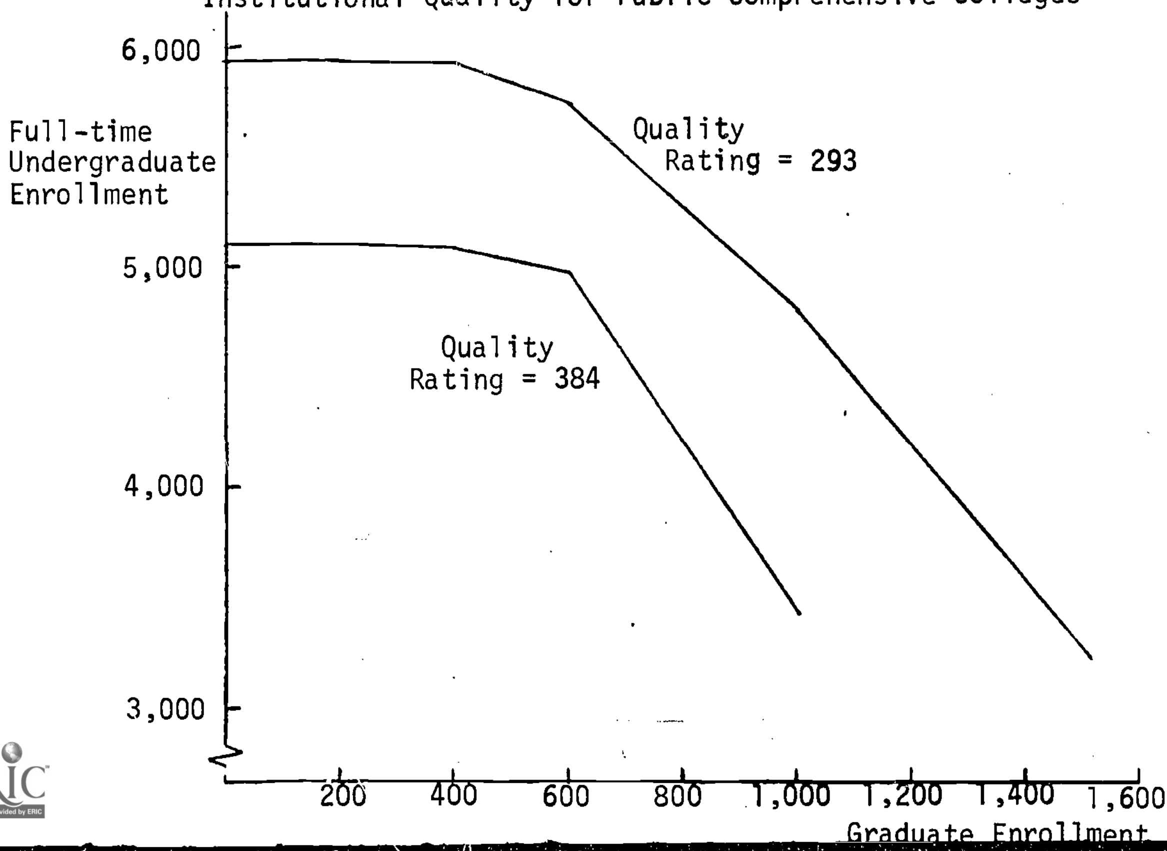


FIGURE 5-7

Graduate - Full-time Undergraduate Transformation Curves by Institutional Quality for Public Comprehensive Colleges



The computational technique developed in Chapter IV has yielded, in considerable detail, a description of the frontier production and cost relationships for several samples of higher education institutions. Productivity relationships between inputs and enrollment variables, substitution relationships between alternative input variables, transformation relationships between different types of enrollment, and various cost relationships have all been shown to depend significantly on input mixes and levels, enrollment mixes and levels, and the values of all the institutional characteristic variables. It should be noted that all of these empirical relationships represent frontier behavior as observed from the samples of institutions rather than average behavior. Also, no functional form restrictions, other than convexity, have been placed on the various production and cost relationships. Many implications of the results for higher education production and cost analysis have been stated throughout this chapter and additional conclusions and implications will be made in the remaining chapters.

VI. SENSITIVITY OF RESULTS TO INDIVIDUAL OBSERVATIONS

A critical aspect of the Farrell approach to production function estimation is the sensitivity of the results to errors in the data. Since the extremes of the data determine the frontier, outliers in the data may strongly influence the estimated, frontier relationships. The desired procedure is to determine extremes in behavior, not extremes due to measurement and reporting errors.

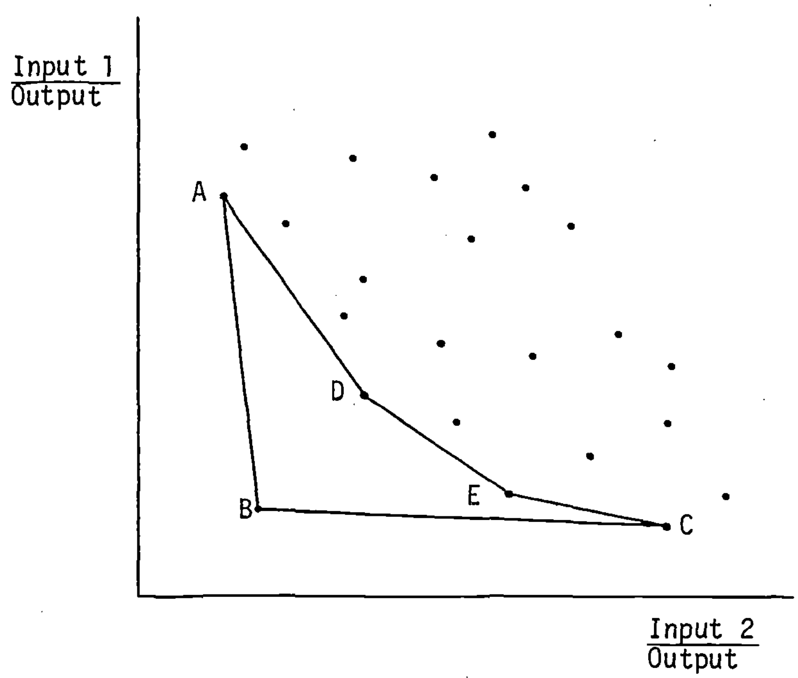
Obviously, instit :ions can have legitimate reasons other than inefficiency for not being on the production and cost frontiers as determined from one year of cross-section data. For example, an institution may be adding new programs, expanding older ones, or engaging in any activity that requires the institution to use more resources than normal given the institution's enrollment and characteristics. Arguing that these cases might cause problems in the empirical analysis makes it even more important to use a frontier analysis rather than an average estimation approach, since the "special situation" institutions are excluded from the determination of the frontier production and cost relationships. It is more likely that non-measurable aspects that affect costs are ones that increase costs rather than decrease costs. That is, excluding reporting errors, it is much more difficult to suggest reasons why an institution would be "super-efficient" because of some certain set of circumstances. The crucial consideration is to make sure that the frontier institutions are not the result of spurious data. Although the computational procedure in this study generates information about the relative efficiency of institutions, caution should be used in labeling specific institutions as being inefficient without a more detailed analysis of their particular situation during the time period studied. The main emphasis of this study is to determine the frontier production and cost relationships as observed from the data rather than to compute efficiency indices for each institution in the sample.

In this chapter, various methods of illustrating the sensitivity of the frontier production and cost relationships are presented. The following sections include a detailed analysis of the data by institution with respect to several key ratios, a listing of the observed institutions that describe some of the frontier relationships, a tabulation of all the frontier institutions, a discussion of the sample dependency of the estimation results, and an examination of the consistency of the results across institutional categories and their consistency with economic theory.

Detailed Data Analysis

If the empirical analysis only involved two or three variables, a graphic display of the data could be used to indicate the location of the frontier institutions with respect to all the other institutions. An illustration of the effect of an outlying observation on a frontier relationship is given in Figure 6-1 for the simple case of two inputs and one output. Since observation B is a considerable distance from the other plotted points, it has an enormous influence on the shape of the isoquant. For the case of several variables, the graphic approach is not feasible, since the data would have to be sliced very thin in order to get down to two dimensions and have all

FIGURE 6-1
Illustration of the Effect of an Outlier Observation



the other variables at roughly the same levels. Some alternatives to the graphic approach are to examine the extremes of several key ratios of variables and to note the amount of "shape" in the calculated, frontier relationships. As shown in Figure 6-1, the effect of observation B is to make the frontier isoquant run parallel to the axes of the graph. This relationship means that changing the level of one input has little or no effect on the level of the other input. The occurrence of this relationship in the empirical results is discussed later in this chapter.

As a means of illustrating the location of the constructed, frontier institutions to other observed institutions with respect to several ratios, Tables 6-1 through 6-7 are presented. The three ratios are total enrollment divided by total faculty, total enrollment divided by classroom space, and total cost divided by total enrollment. The ratios are computed for each institution and the ten largest values for each ratio are listed in the tables. Also, the average over the five largest and ten largest values for each ratio are given along with the ratios constructed by dividing average total enrollment by the minimum values for senior faculty and classroom space and by dividing the minimum total cost by average total enrollment. For example, the number of senior faculty resulting from minimizing senior faculty with all other variables at their means is used in calculating the "minimum" value shown in the tables for the total enrollment to total faculty ratio. The last value shown in each column is the result of using the averages over all the institutions in each sample for each variable in the ratio.

As Tables 6-1 through 6-7 show, several observed institutions

TABLE 6-1

Ratio Analysis for Public Universities

Rank	Total Enrollment / Total Faculty	Total Enrollment / Classroom Space	Total Cost / Total Enrollment
1	25.5	.159	887
2	23.3	.149	974
3	21.5	.132	1,022
4	21.5	.117	1,062
5	21.0	.114	1,096
6	20.5	.111	1,234
7	20.0	.109	1,280
8	19.7	.106	1,346
9	19.6	.105	1,374
10	18.9	.105	1,686
Average of Top 5	22.6	.134	1,008
Average of Top 10	21.2	.121	1,196
Minimum	Senior Faculty: 16.6	Classroom Space: .116	Cost: 1,757
Average	14.4	.092	2,415

TABLE 6-2

Ratio Analysis for Private Universities

Rank	Total Enrollment / Total Faculty	Total Enrollment / Classroom Space	Total Cost / Total Enrollment
1	27.1	.479	812
2	18.9	.134	1,107
3	18.4	.126	1,245
4	17.2	.120	1,493
5	16.1	.120	1,725
6	16.1	.108	1,935
7	16.1	.100	2,224
8	14.8	.098	2,311
9	13.8	.095	2,378
10	12.4	.093	2,643
Average of Top 5	19.5	.196	1,276
Average of Top 10	17.1	.147	1,787
Minimum	Senior Faculty: 13.3	Classroom Space: .128	Cost: 2,415
Average	7.7	.083	5,348

TABLE 6-3

Ratio Analysis for Public Comprehensive Colleges

Rank	Total Enrollment / Total Faculty	Total Enrollment / Classroom Space	Total Cost / Total Enrollment
1	37.4	.207	504
2	30.8	.197	526
3	30.7	.179	630
4	30.0	.166	672
5	29.0	.150	727
6	28.0	.147	742
7	27.4	.145	771
8	26.7	.142	775
9	25.9	.140	775
10	25.9	.137	798
Average of Top 5	31.6	.180	612
Average of Top 10	29.2	.161	692
Minimum	Senior Faculty: 27.8	Classroom Space: .141	Cost: 670
Average	20.5	.081	1,120

TABLE 6-4

Ratio Analysis for Private Comprehensive Colleges

Rank	Total Enrollment / Total Faculty	Total Enrollment / Classroom Space	Total Cost / Total Enrollment
1	47.4	.191	353
2	43.9	.186	674
3	42.5	.157	703
4	32.7	.144	718
5	31.9	.136	742
6	31.2	.136	775
7	30.2	.133	802
8	29.4	.126	816
9	29.1	.123	857
10	28.5	.121	878
Average of Top 5	32.1	.132	651
Average of Top 10	39.7	.163	731
Minimum	Senior Faculty: 34.7	Classroom Space: .145	Cost: 687
Average	18.9	.081	1,319

TABLE 6-5

Ratio Analysis for Public Limited Comprehensive Colleges

Rank	Total Enrollment / Total Faculty	Total Enrollment / Classroom Space	Total Cost / Total Enrollment
1	34.0	.186	643
2	28.7	.160	653
3	27.4	.154	676
4	25.1	.133	793
5	24.6	.125	811
6	24.4	.118	845
7	24.4	.115	854
8	22.6	.106	869
9	22.1	.100	924
10	22.1	.097	931
Average of Top 5	28.0	.152	715
Average of Top 10	25.5	.130	800
Minimum	Senior Faculty: 25.1	Classroom Space: .128	Cost: 787
Average	18.9	.061	1,120

TABLE 6-6

Ratio Analysis for Private Highly Selective Liberal Arts Colleges

Rank	Total Enrollment / Total Faculty	Total Enrollment / Classroom Space	Total Cost / Total Enrollment
1	18.4	.089	989
2	18.4	.087	1,181
3	18.1	.079	1,215
4	17.4	.077	1,223
5	17.3	.074	1,297
6	16.3	.074	1,385
7	15.6	.074	1,388
8	15.1	.072	1,404
9	15.0	.070	1,431
10	14.7	.068	1,437
Average of Top 5	17.9	.081	1,181
Average of Top 10	16.6	.076	1,295
Minimum	Senior Faculty: 15.4	Classroom Space: .072	Cost: 1,538
Average	12.0	.040	2,381

TABLE 6-7

Ratio Analysis for Private Less Selective Liberal Arts Colleges

Rank	Total Enrollment / Total Faculty	Total Enrollment / Classroom Space	Total Cost / Total Enrollment
1	30.6	.195	556
2	28.6	.134	668
3	28.1	.128	684
4	27.7	.124	782
5	26.9	.120	783
6	24.9	.104	787
7	24.4	.096	789
8	24.2	.094	799
9	24.1	.092	804
10	24.0	.090	877
Average of Top 5	28.4	.140	695
Average of Top 10	26.4	.118	754
Minimum	Senior Faculty: 24.7	Classroom Space: .119	Cost: 841
Average	14.8	.041	1,439

have values for these ratios that are much larger (smaller in the case of the total cost ratio) than the "minimum" ratio. For all the institutional categories, the "minimum" total enrollment to total faculty ratio is less than the average of the five largest and the ten largest student-faculty ratios. A similar result holds for the total enrollment to classroom space ratio, except that in the case of the private less selective liberal arts category the minimum ratio is slightly larger than the average of the ten largest ratios. For the total cost to total enrollment ratios, the "minimum" value is always greater than the average of the five smallest cost per student values, but in three cases (public comprehensives, private comprehensives, and public limited comprehensives) the "minimum" value is slightly less than the average of the ten smallest ratios.

These results imply that the frontier relationships are not solely determined by the institutions with the extreme values for these three ratios. For example, the constructed institutions with average enrollment, average characteristics, and minimum cost do not have the smallest cost per student of all observed institutions. The observed institutions with larger total enrollment to total faculty and total enrollment to classroom space and with smaller cost per student must have less "expensive" mixes of enrollment and/or a less expensive set of characteristics. More information on this behavior is given below. The important point here, as far as the sensitivity of the results, is that the frontier relationships are not simply determined by some extreme combination of variables for one or two institutions. It takes several observed institutions to determine one of the constructed, frontier institutions, as illustrated later in this chapter.

To go one step further in looking at the actual data and at how close it fits into the calculated frontier relationships, two components of the total enrollment to total faculty ratio are plotted in Figures 6-2 through 6-8. As before, these graphs show the constructed frontier institutions to be quite some distance from the observed extreme observations. Again this emphasizes that the extreme points have less "expensive" characteristic variable sets and/or enrollment mixes, although in these graphs the undergraduate-graduate mix for the university groups and the undergraduate part-time full-time mix for the other colleges are being appropriately accounted for. In these graphs, only the ten institutions with the largest student-faculty ratios, the institutions in the solution basis for minimum senior faculty, the constructed average institution with minimum senior faculty, and the average institution are plotted.

To go into any further detail with graphic data analysis is extremely cumbersome. As mentioned before, the computational method being used is, in fact, a means of doing the same thing arithmetically. However, the graphic and tabular results given so far in this chapter have indicated that the results generated by the linear programming method are not simply determined by one or two spurious observations but represent a complex balance of all the variables for many of the institutions in the analysis.

The "Optimal Basis" Institutions

Another way of determining the sensitivity of the generated results to individual observations is to analyze the number and composition of institutions in the solution basis of each linear

FIGURE 6-2

Graphic Display of Selected Public Universities

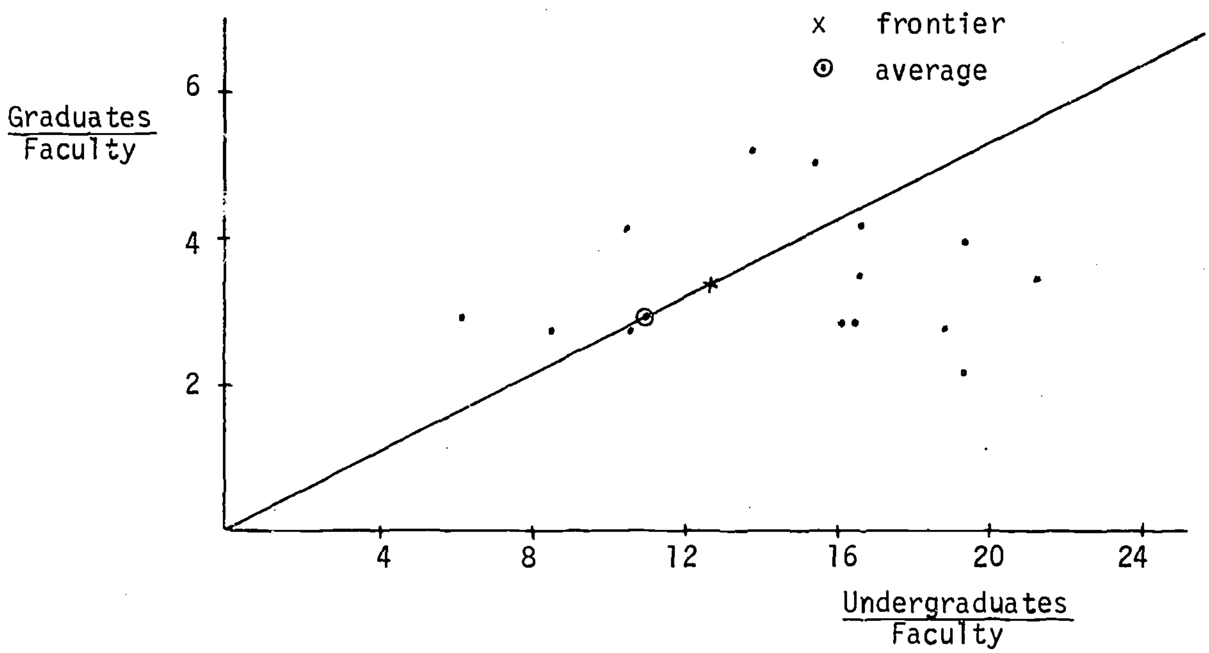


FIGURE 6-3

Graphic Display of Selected Private Universities

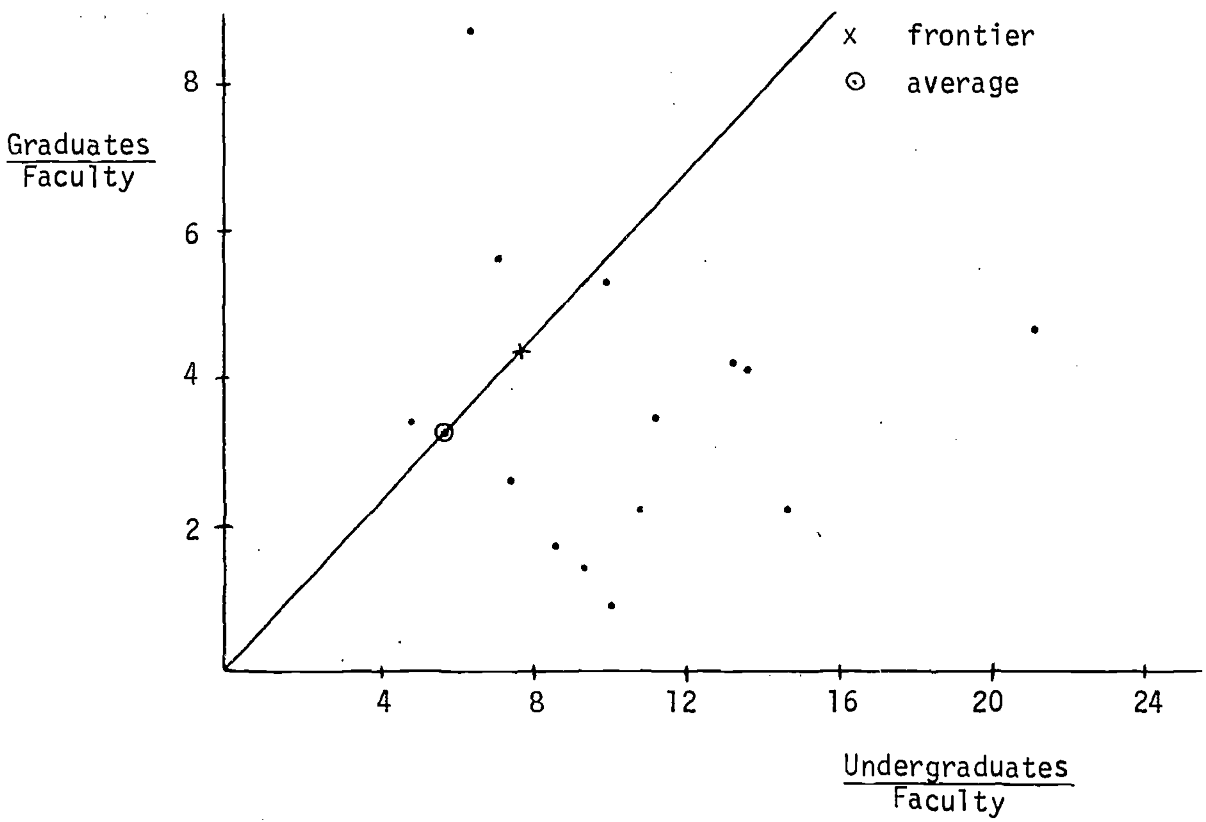


FIGURE 6-4

Graphic Display of Selected Public Comprehensive Colleges

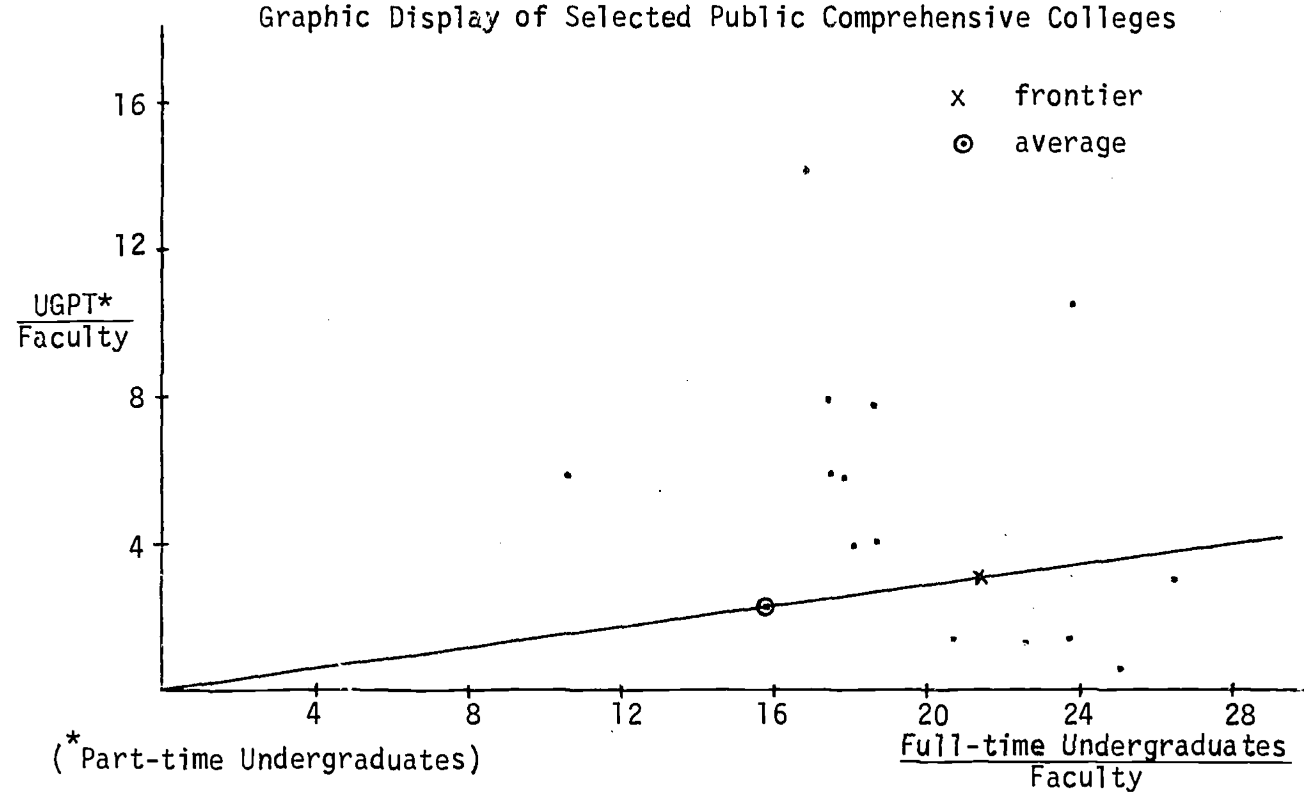


FIGURE 6-5

Graphic Display of Selected Private Comprehensive Colleges

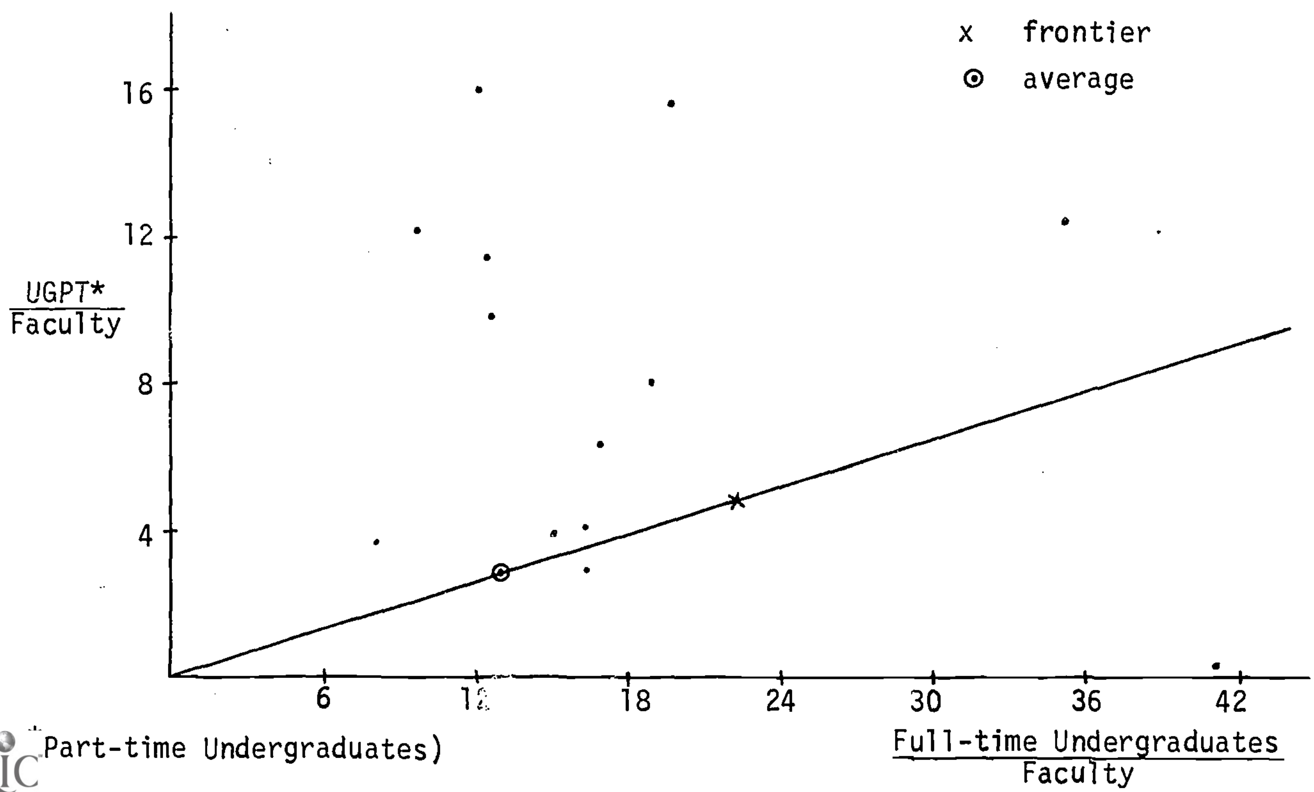


FIGURE 6-6

Graphic Display of Selected Public Limited Comprehensive Colleges

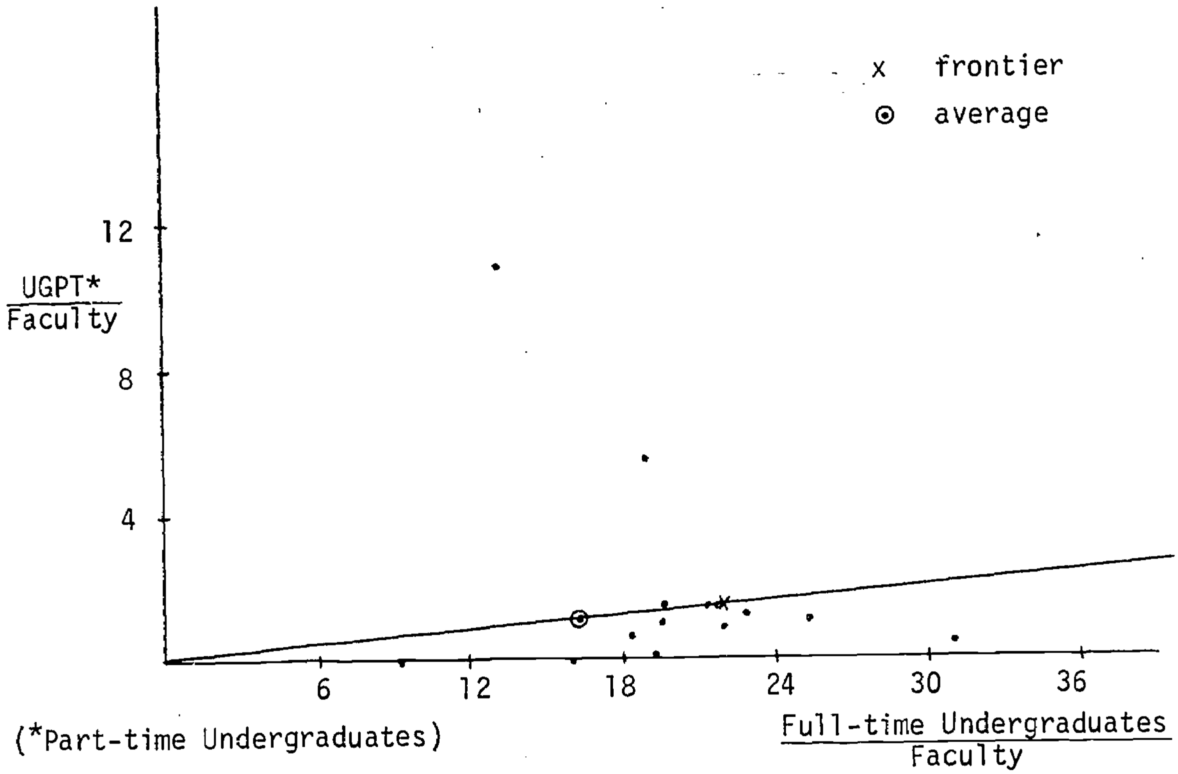


FIGURE 6-7

Graphic Display of Selected Private Highly Selective Liberal Arts Colleges

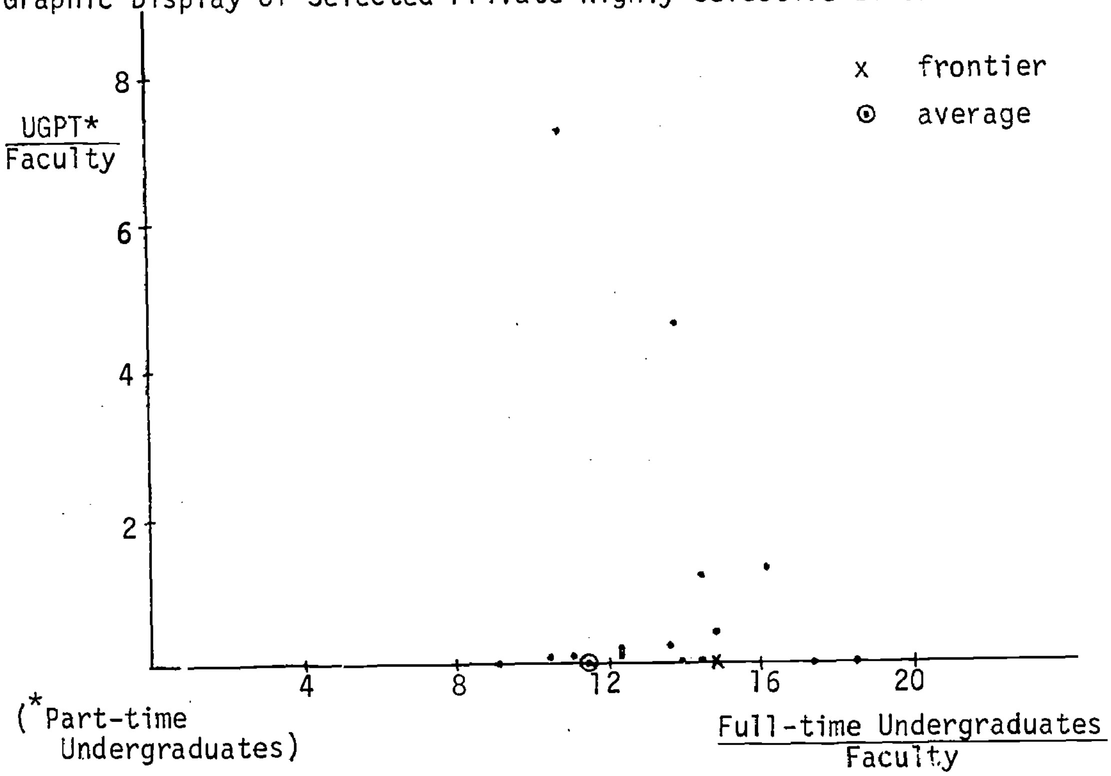
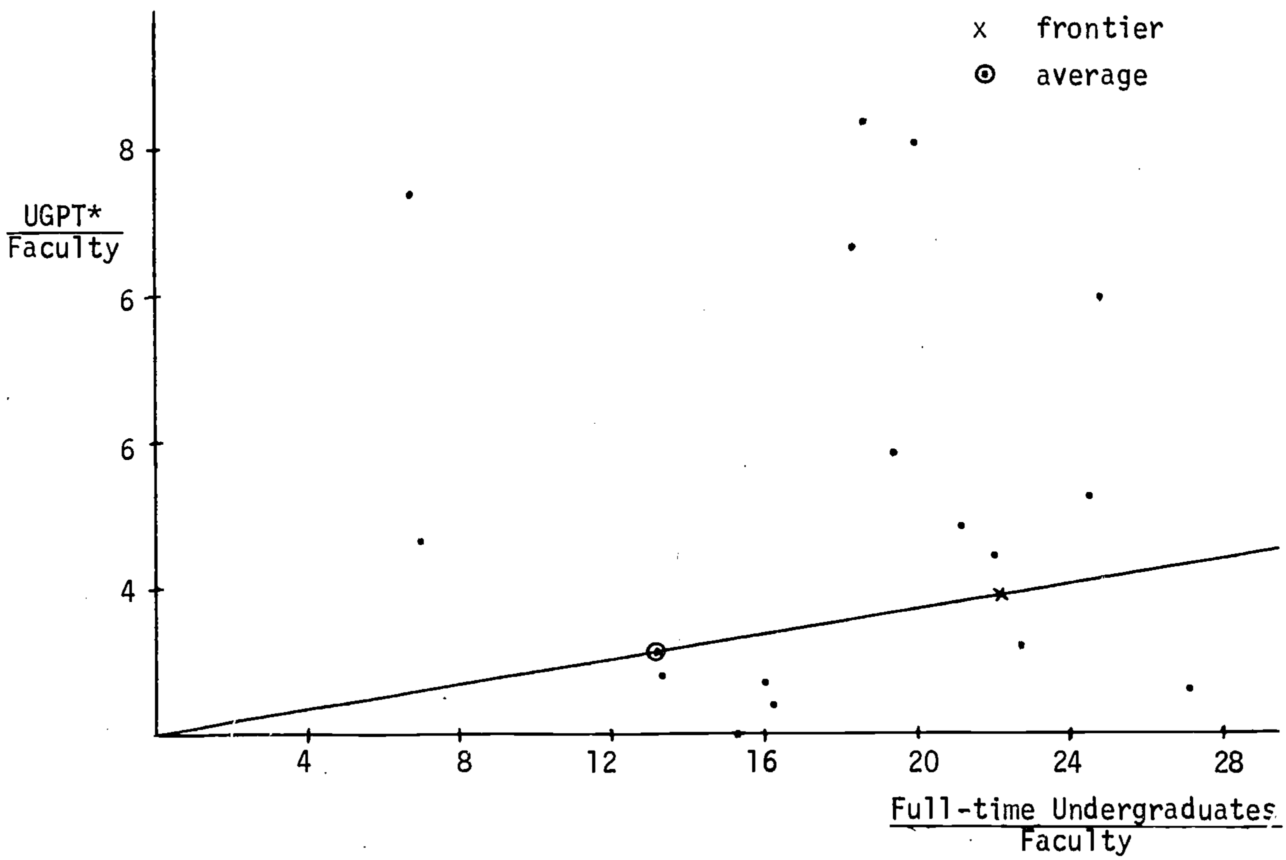


FIGURE 6-8

Graphic Display of Selected Private Less Selective Liberal Arts Colleges



(*Part-time Undergraduates)

programming problem solved to determine a specific point on the production or cost frontier. This procedure provides information to test the dependency of specific points on the frontier to individual observations. Tables 6-8 through 6-14 show for each institutional category the observed institutions that are used in constructing the frontier institution with averages for all variables except the one variable in the linear programming objective function. As the column headings indicate, the bases for four LP models are illustrated: minimizing senior faculty, minimizing classroom space, minimizing total cost, and maximizing full-time undergraduates. For each LP model, the identification number for each institution included in the construction of the frontier is given along with the weight used in the summation (the z_t's as defined in Table 4-1). It should be noted that the weights do not necessarily add to one and that the magnitude of the weights must be interpreted in conjunction with the relative size of the respective institutions. The proportional effect of one institution on the constructed, frontier institution is determined by the product of the weight (z_t) and the magnitude of the institution's objective function variable.

Several interesting points should be noted from Tables 6-8 through 6-14. First, several observed institutions are required in order to construct just one of the frontier points. The range is from five to twelve institutions with an average of 9.2 institutions. Second, the same set of observations does not determine all four efficient points within an institutional category. For those institutions that do reappear, the weights fluctuate substantially across the four LP models. Third, the magnitudes of the weights for each LP solution

TABLE 6-8

Composition of Frontier Institutions - Public Universities

Sequence	Senior Faculty		Classroom Space		Total Cost		Full-time Undergraduates	
	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight
1	3	.098	8	.002	7	.168	2	.051
2	5	.001	10	.181	9	.005	5	.011
3	7	.080	11	.069	10	.114	8	[illegible]
4	9	.009	13	.151	15	.005	10	.075
5	10	.076	14	.111	24	.073	13	.135
6	15	.036	19	.135	26	.045	14	.011
7	20	.194	24	.001	32	.401	24	.100
8	26	.092	27	.427	35	.267	26	.115
9	35	.367					27	.036
10							34	.106
11							35	.261

Summary: (a) 19 different institutions.

(b) institution #10 is common to all four models.

TABLE 6-9

Composition of Frontier Institutions - Private Universities

Sequence	Senior Faculty		Classroom Space		Total Cost		Full-time Undergraduates	
	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight
1	2	.075	3	.106	1	.002	3	.053
2	4	.267	5	.110	2	.024	4	.143
3	7	.067	7	.128	3	.067	5	.177
4	12	.217	12	.179	11	.041	7	.283
5	20	.163	14	.117	12	.072	12	.377
6	25	.036	24	.267	16	.014	14	.032
7	28	.398	29	.073	22	.225	29	.067
8					28	1.151	31	.053

Summary: (a) 17 different institutions.

(b) institution #12 is common to all four models.

TABLE 6-10

Comparison of Frontier Institutions - Public Comprehensive Colleges

Sequence	Senior Faculty		Classroom Space		Total Cost		Full-time Undergraduates	
	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight
1	45	.022	11	.031	2	.035	8	.013
2	64	.024	13	.017	11	.011	11	.044
3	67	.186	24	.059	67	.601	30	.001
4	69	.391	30	.430	81	.011	45	.031
5	85	.046	45	.006	85	.087	64	.031
6	88	.066	65	.045			67	.375
7	92	.006	67	.134			80	.261
8	93	.140	70	.009			85	.033
9			71	.018			88	.262
10							90	.128

Summary: (a) 20 different institutions.

(b) institution #67 is common to all four models.

TABLE 6-11

Composition of Frontier Institutions - Private Comprehensive Colleges

Sequence	Senior Faculty		Classroom Space		Total Cost		Full-time Undergraduates	
	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight
1	11	.008	1	.009	3	.020	2	.027
2	12	.010	2	.037	12	.003	13	.009
3	17	.005	11	.058	14	.222	14	.109
4	21	.068	22	.098	22	.084	23	.353
5	24	.434	28	.015	30	.740	29	.047
6	38	.115	41	.039	58	.005	38	.062
7	41	.094	46	.080	64	.015	47	.020
8	47	.017	49	.183			49	.291
9	82	.645	51	.492			64	.067
10			63	.033			82	.717

Summary: (a) 25 different institutions.

(b) no institution is common to all four models.

TABLE 6-12

Composition of Frontier Institutions - Public Limited Comprehensive Colleges

Sequence	Senior Faculty		Classroom Space		Total Cost		Full-time Undergraduates	
	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight
1	5	.295	3	.034	3	.222	5	.393
2	14	.054	14	.023	13	.092	14	.029
3	16	.122	21	.008	14	.041	16	.010
4	18	.386	22	.056	22	.069	18	.294
5	21	.011	29	.305	33	.157	21	.006
6	22	.032	33	.047	36	.023	22	.032
7	33	.064	35	.432	46	.043	29	.066
8	36	.026	36	.168	55	.246	36	.165
9	47	.003	46	.043			45	.307
10	52	.011	52	.005			46	.040
11			57	.141			47	.023
12			59	.016			57	.090

Summary: (a) 19 different institutions.

(b) institutions #14, 22, 36 are common to all four models.

TABLE 6-13

Composition of Frontier Institutions - Private Highly Selective Liberal Arts Colleges

Sequence	Senior Faculty		Classroom Space		Total Cost		Full-time Undergraduates	
	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight
1	2	.292	6	.248	2	.088	2	.233
2	6	.021	9	.185	4	.055	6	.043
3	8	.051	18	.019	8	.005	8	.060
4	13	.027	26	.061	18	.060	13	.011
5	15	.283	45	.091	22	.029	15	.354
6	19	.015	53	.054	49	.649	19	.048
7	20	.065	61	.047	50	.061	20	.011
8	35	.064	68	.022	60	.021	35	.099
9	68	.044	79	.023	61	.062	50	.111
10	79	.022	80	.272	79	.014	79	.019
11							80	.077

Summary: (a) 22 different institutions.
(b) institution #79 is common to all four models.

TABLE 6-14

Composition of Frontier Institutions - Private Less Selective Liberal Arts Colleges

Sequence	Senior Faculty		Classroom Space		Total Cost		Full-time Undergraduates	
	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight	Institution Number	Weight
1	10	.010	12	.033	10	.020	10	.007
2	16	.022	22	.026	16	.021	16	.040
3	30	.168	84	.078	78	.717	19	.010
4	84	.216	131	.004	131	.004	30	.254
5	110	.015	134	.424	178	.130	84	.460
6	173	.037	154	.006	212	.124	131	.006
7	180	.044	211	.048	231	.048	173	.047
8	190	.230	212	.002	262	.069	180	.085
9	191	.052	237	.238			190	.135
10	231	.021	262	.038			231	.051
11	259	.171					259	.127
12							262	.047

Summary: (a) 23 different institutions.

(b) No institutions are common to all four models.

vary drastically across the institutions included in the final basis. In only a very few cases does one institution account for over 50% of the value of the LP objective function variable. Fourth, for these four efficient points within each institutional category, it takes from 17 to 25 institutions to determine the frontier points with an average of 20.7. Fifth, only about half of the "basis" institutions correspond to institutions given in the previous section with the largest ratios. Furthermore, some of the institutions included in the solution basis rank relatively low with respect to some of these ratios. All of the basis institutions, however, are themselves on the frontier.

The above results also imply that the frontier relationships are not being determined simply by one or two spurious observations. Many institutions in many different combinations are needed in order to describe various points on the production and cost frontiers.

An additional way to test the sensitivity of the results to individual observations was used by C. P. Timmer [1970] for another method of frontier estimation. His approach to the problem was to simply delete the observations that were used in constructing the frontier, re-estimate and continue the process until the estimates stabilize. A similar procedure could be implemented here; however, there is not too much to be gained from doing such an analysis. If deleting an observation does not change the relationship very much, the observation is not contributing much to the results. However, if deleting the observation results in a large shift in the results, the decision of whether or not to discard the observation must be made. Where is the cutoff of "level of significance" that determines if an observation is discarded or not? Throwing away useful information is as serious as incorporating spurious information. The description of the data given in the previous sections of this chapter and the information presented below provide some evidence that spurious data are not strongly influencing the calculated, frontier production and cost relationships.

Number of Efficient Institutions

An additional means of determining the sensitivity of the generated results to individual observations is to look at the number of observed institutions required to describe all of the frontier cost relationships. The cost analysis (based on 13 variables) is used rather than the production analysis (based on 19 variables), since it has fewer dimensions; any of the implications of this analysis are conservative estimates of implications for a similar analysis of the production relationships. Table 6-15 shows by institutional category the total number of institutions in each sample, the number of institutions on the frontier, and the percent of institutions on the frontier. As the table illustrates, the percentage of frontier institutions is quite large; in all cases the percentage is greater than 50%. This large percentage is the result of the wide diversity in enrollment structures and characteristic variables observed within each of the institutional categories. Also, the large percentages imply that the frontier relationships are not being determined from only a few observations. If only five or ten percent of the observations were located on the frontier, concern might be expressed about the reliability of the results being based on so few observed data points.

TABLE 6-15

Number of Frontier Institutions

Institutional Type	Total Number in Sample	Number on Frontier	Percent on Frontier
Public Universities	37	31	84
Private Universities	31	24	77
Public Comprehensive Colleges	105	58	55
Private Comprehensive Colleges	92	48	52
Public Limited Comprehensive Colleges	59	37	63
Private Highly Selective Liberal Arts Colleges	81	44	54
Private Less Selective Liberal Arts Colleges	269	*	*

*Too expensive to compute.

Sample Dependency

The results from this frontier analysis are obviously sample dependent and caution must be stressed in extrapolating these results to all institutions. It should be noted, however, that if there are no gross measurement or reporting errors, the calculated production and cost frontiers are conservative estimates of the "true" frontiers. This relationship follows, since the only way additional observations can affect the results will be if the new observations are more efficient. The frontier can only be moved in one direction by adding observations.

Consistency of Results

As the descriptive results presented in Chapter V indicate, the various production and cost relationships are consistent across the institutional categories. These patterns provide an indirect indication of the sensitivity of the frontier relationships to individual observations. If particular "spurious" observations were contaminating the determination of the frontier relationships, the relationships would most likely not be similar across the different samples of institutions. It is highly unlikely that random "extreme" observations would occur in all the samples in a manner that leads to similar relationships.

The empirical results are also consistent with economic theory. The input isoquants and the enrollment transformation curves have the correct shape, and the characteristic variables behave as originally specified. Also, the various frontier production and cost relationships have a considerable amount of "shape." That is, changes in one variable over a fairly wide range of values corresponds to changes in other variables along the production and cost frontiers. This observed behavior is encouraging for the estimation method, since it implies that "outlier" observations like B in Figure 6-1 are not contaminating the results.

VII. INSTITUTIONAL COST BEHAVIOR

The descriptive results presented in Chapter V by no means exhaust the information about the frontier cost and production behavior that can be generated from the cross-sectional data with the computational procedure used in this study. The purpose of the present chapter is to use the methodology developed in the earlier chapters and the institutional data to study several aspects of institutional cost behavior. These examples of extending the basic analysis illustrate the potential of this empirical approach for further research.

Variations in Costs Per Students

Many empirical studies and reports on higher education have illustrated the extreme variations that exist in costs per student across samples of institutions (e.g., H. Jenny and G. Wynn, [1970, 1972]; Columbia Research Associates, [1971]; J. Powell and R. Lamson, [1972]; and the Carnegie Commission, [1972]). From these cost per student variations the implication often made is that these large variations are primarily the result of inefficiencies. If all institutions were operating efficiently, then the variations in costs per student would be drastically reduced. The other explanation for these large variations is that the enrollment mixes and characteristics of the institutions vary considerably, and the costs are simply the result of more (or less) expensive enrollment mixes and institutional characteristics. As a means of determining the extent to which both of these hypotheses are correct with the data and analytical framework of this study, the variance of total educational and general expenditures per student is computed for each institutional category. In addition, the variance of cost per student for the institutions lying on the cost frontier and the sum of squared deviations of cost per student from that observed for the non-frontier institutions to the cost frontier are computed and the results are given in Table 7-1. As expected, the mean cost per student for the frontier institutions is less than the mean cost per student for all the institutions in each institutional category. The standard deviation and the coefficient of variation of the cost per student for the frontier institutions and all institutions are roughly the same for all categories of institutions. This implies that efficiency relative to the computed frontier is not the main determinant of the variation in cost per student. The variation in cost per student for the frontier institutions is due to enrollment mix and institutional characteristic variations. The average deviation from the frontier is roughly equal to the standard deviation of cost per student for the frontier institutions in each institutional category except for the public universities, where the average deviation is less than the standard deviation. Therefore, the average deviation of the non-frontier institutions from the observed cost frontier is roughly one standard deviation from the frontier. The standard deviation of the deviations of the non-frontier institutions from the frontier is always less than the standard deviation of cost per student for the frontier institutions. Since the mean deviation is small relative to the average cost per student, the coefficient of variation for the deviations is always greater than the coefficient of variation for the cost per student for the frontier

TABLE 7-1

Cost per Student Variations

Institutional Type	All Institutions			Frontier Institutions			Deviations from Frontier		
	μ*	σ**	C.V.†	μ	σ	C.V.	μ	σ	C.V.
Public Universities	2,499	1,070	0.43	2,206	1,164	0.53	703	636	0.90
Private Universities	4,609	2,911	0.63	3,960	2,390	0.60	2,784	2,365	0.85
Public Comprehensive Colleges	1,144	311	0.27	965	324	0.34	371	255	0.69
Private Comprehensive Colleges	1,434	563	0.39	1,198	438	0.37	431	322	0.75
Public Limited Comprehensive Colleges	1,172	277	0.24	1,071	272	0.25	271	273	1.01
Private Highly Selective Liberal Arts Colleges	2,393	831	0.35	2,026	825	0.41	802	551	0.69

* μ = mean

** σ = standard deviation

† Coefficient of Variation = σ/μ

institutions.

These results indicate that, at least relative to the cost frontier computed from the observed cross-section of institutions, the large variation in cost per student is not due primarily to inefficiency. The frontier institutions exhibit a large variation in costs per students due to alternative enrollment mixes and differing institutional characteristics. These results also indicate the problems of setting one target level of cost per student for all institutions in order to increase the efficient use of resources in higher education. As the above simple calculations and the descriptive results in Chapter V show, cost per student varies considerably as the result of different enrollment mixes and institutional characteristics.

Public Versus Private Institutions

Although public and private institutions have different goals and objectives, it is interesting to compare the public universities to the private universities and the public comprehensive colleges to the private comprehensive colleges within the framework of this study. Since the average per student cost is usually much higher for the private groups of institutions (see Table 5-17), the implication often made is that private institutions are more expensive. The purpose of this section is to compare similar groups of public and private institutions and see if public and private costs differ when enrollment mix and institutional characteristics are controlled.

To perform this comparison, a hypothetical university was constructed with the enrollment and characteristic variables equal to the average of the means of the variables for the public and private universities. Using the public university data and then the private university data, linear programming problems were solved to determine the minimum cost for an institution with these average public-private characteristics. The results are given in Table 7-2, and the minimum cost solution to the LP using the public university data is much larger ($4,180 per student) than the solution using the private university data ($1,845 per student). A similar set of calculations is performed for the public and private comprehensive colleges, and the results are shown in Table 7-3. Again the public least-cost solution ($850 per student) is greater than the private least-cost solution ($804 per student).

These results are rather surprising. Even though the average costs per student for private institutions are generally much higher than the average costs per student for public institutions, it appears that for certain enrollment combinations and characteristic sets private institutions have a lower cost per student. This implies that the higher average cost per student for private colleges and universities is the result of a more "expensive" mix of enrollment and institutional characteristics. If private institutions offered the same package as the public institutions, the average cost per student would probably be the same as (or even lower than) the public institutions.

Under-utilization of Private Institutions

The financial squeeze on higher education has hit the private institutions especially hard. As illustrated in Table 7-4, enrollment for all of higher education has been growing, although the rate of

TABLE 7-2

Public - Private Cost Comparison - Universities

Variable	Public Average	Public Solution	Midpoint	Private Solution	Private Average
Gourman Quality Rating	453		500		546
Percent Science Degrees	21		23		25
Enrollment Retention	97		97.5		98
Institutional Scale	4,650		4,842		5,034
Number of Fields	125		107		90
Enrollment Growth Rate	7.6		5.7		3.9
Percent Research Revenues	15.0		21.7		28.4
Percent Public Service Expenditures	5.4		3.5		1.5
Part-time Undergraduates	1,041		802		563
Full-time Undergraduates	9,539		7,500		4,073
Graduate Students	2,697		2,631		2,565
Specialized Enrollment	498		588		678
Cost ($ Million)	24.21	48.25		21.24	19.08
Cost/Student	$1,760	$4,180		$1,845	$2,420

TABLE 7-3

Public - Private Cost Comparison - Comprehensive Colleges

Variable	Public Average	Public Solution	Midpoint	Private Solution	Private Average
Gourman Quality Rating	366		372		379
Percent Science Degrees	12		13.5		15
Enrollment Retention	80		82.5		85
Institutional Scale	903		748		592
Number of Fields	42		36		30
Enrollment Growth Rate	10.0		6.5		3.0
Percent Research Revenues	1.6		1.7		1.8
Percent Public Service Expenditures	1.4		1.25		1.1
Part-time Undergraduates	615		538		461
Full-time Undergraduates	4,335		3,235		2,136
Graduate Students	521		459		397
Specialized Enrollment	158		139		119
Cost ($ Million)	3.77	3.73		3.51	2.43
Cost/Student	$ 670	$850		$804	$ 782

TABLE 7-4

Public and Private Total Enrollment, 1965-70

Year	Public Total Enrollment (000)	Public Ratio*	Private Total Enrollment (000)	Private Ratio*
1965	3,970	1.00	1,951	1.00
1966	4,349	1.10	2,041	1.05
1967	4,816	1.21	2,096	1.07
1968	5,431	1.37	2,082	1.07
1969	5,840	1.47	2,077	1.06
1970	6,371	1.60	2,127	1.09

*Ratio = (total enrollment/1965 total enrollment).

SOURCE: Carnegie Commission on Higher Education staff.

growth has declined in recent years, while for the private institutions enrollment actually declined between 1967 and 1970. A closer look at the frontier production and cost relationships given in Chapter V suggest that for higher education as a whole it would take fewer resources if more of the increases in enrollment were absorbed by private institutions rather than almost completely by public institutions, as has been the trend. The marginal product relationships between full-time undergraduates, senior faculty, and classroom space, as given in Table 5-11, indicate, with few exceptions, that the marginal increase in enrollment for given increases in faculty and classroom space is greater than the average productivity of faculty and classroom space for full-time undergraduates for the private institutions. The public institutions, however, exhibit the marginal increase to be less than the average productivity. The marginal productivity of classroom space for undergraduate enrollment (above the average level of classroom space) is always greater for the private institutions than the corresponding marginal productivities for the public institutions. Although the marginal productivity of faculty for undergraduate enrollment is usually less for the private institutions, this relationship is primarily due to the more "expensive" enrollment mix and institutional characteristics of the private institutions. The scale effects on the production behavior given in Table 5-14 also indicate that the private institutions can gain more from increasing their size than can the public institutions. Similarly, the effect of the enrollment growth on the minimum level of senior faculty (shown in Table 5-16) is much stronger for the private institutions than for the public institutions. Low-growth rates of enrollment correspond to higher levels of senior faculty. Also, note that high growth rates do not lead to any advantages over average growth rates. That is, the primary effect of the enrollment growth variable is from low values to average values. Extremely high growth rates do not lead to any economies. Many of the public institutions have experienced these high growth rates during the late 1960's while most private institutions grew at much slower rates.

These results imply that if no non-economic barriers existed, if would be more conservative of national resources if the private institutions absorbed more of the enrollment increases rather than if the public institutions did so. In terms of a policy this would require federal and state monies to be given to private institutions rather than public institutions and the general stature of the private institutions would have to change. Although these results run counter to many non-economic factors, it is interesting to note that the use of under-utilized private institutions to take the growth strain off of public institutions could lead to a more efficient use of national and state resources in higher education.

Variable Interaction Effects

All of the descriptive results presented in Chapter V were based upon the "average" institution. The effects of changes in one variable on the frontier level of another variable were studied in great detail. The purpose of this section is to illustrate for the highly selective liberal arts colleges the interaction effects of changing more than one variable at a time. As discussed in Chapter IV, regression estimation methods often leave out interaction effects between variables, or at best they are rigidly specified. With Farrell's method, none of the interaction effects are left out, nor are they given a prior specification. As with the other descriptive results, each relationship has to be explicitly evaluated. Some results are presented in Table 7-5 to illustrate the effect of changes in several of the characteristic and enrollment variables on the minimum cost per student as observed from the data. None of the results are surprising. The basic relationships between minimum cost per student and the characteristic variables continue to hold. The magnitudes of the changes in minimum cost per student corresponding to changes in one variable (i.e., scale) vary slightly as changes in other variables (i.e., quality, percent science) are made. Also, the interaction between certain pairs is stronger than between other variables. For example, scale has a larger effect on the relationship between the number of fields and average costs than on the relationship between quality and average costs. However, the direction of the relationships will always stay the same; just using the descriptive results based on the average institution provide fairly accurate information about the general effects of changes in osts per student resulting from changes in other variables. Obviously, it is impossible to compute every possible combination of variables, but it is always easy to trace through certain combinations that are interesting or relevant to the evaluation of certain policy proposals (i.e., the public-private comparison performed earlier in this chapter).

TABLE 7-5

Variable Interaction Effects on Cost Per Student -
Highly Selective Liberal Arts Colleges

Gourman Quality Rating	Institutional Scale		
	302	431	604
388	$1,462	$1,450	$1,450
422	1,580	1,540	1,530
464	*	1,700	1,680

Number of Fields	Institutional Scale		
	302	431	604
22	$1,580	$1,540	$1,530
26	1,730	1,590	1,562
31	*	1,865	1,807

Number of Fields	Gourman Quality Rating		
	338	422	464
18	$1,415	$1,530	$1,690
22	1,450	1,540	1,700
26	1,570	1,590	1,720

Number of Fields	Percent Science Degrees		
	12.3	15.0	24.6
18	$1,490	$1,530	$1,740
22	1,490	1,540	1,810
26	1,530	1,590	2,040

*Infeasible combination of variables.

TABLE 7-5 (continued)

Percent Science Degrees	Gourman Quality Rating		
	338	422	464
12.3	$1,395	$1,490	$1,670
15.0	1,450	1,540	1,700
24.6	1,810	1,810	1,970

Institutional Scale	Percent Science Degrees		
	12.3	15.0	24.6
302	$1,540	$1,580	$1,950
431	1,490	1,540	1,810
604	1,485	1,530	1,790

Cost Hypotheses

The behavior of per student or unit costs at higher education institutions can be affected either by changes in the set of input variables or by changes in the enrollment and characteristic variables. Changes in the latter imply a change in <u>what</u> an institution is producing, while changes in the mix and level of inputs imply a change in <u>how</u> an institution produces its outputs.

With respect to the <u>how</u> of institutional production, Bowen and Douglass [1971] suggest several ways in which colleges could change their costs by manipulating their inputs. These cost hypotheses are listed below along with a brief discussion of empirical results from this study that lend support to these cost behavior suggestions.

(1) <u>Substitution of low-cost labor for high-cost labor</u>. The results given in Table 5-12 illustrate that to a limited degree junior faculty are substituted for senior faculty on the production frontier. This substitution occurs mainly at very low levels of junior faculty and relatively high levels of faculty. That is, once the junior to senior faculty ratio reaches a certain point (this level varies by institutional type), an increase in the number of junior faculty does not correspond to a decrease in senior faculty. For all institutional categories except the public limited comprehensive colleges the initial substitution rates are such that substituting junior faculty for senior faculty would lead to reduced costs. A similar relationship exists between senior faculty and general administration-library personnel on the production frontier. The least-cost results in Table 5-22 also indicate that some substitution occurs between senior faculty and general administration-library personnel in response to

changes in relative unit prices for these two types of labor inputs.

(2) Increase intensity of labor usage. As shown in Table 5-2, the number of senior faculty can be reduced drastically, while the same level and mix of enrollment and the same set of institutional characteristics is maintained. This behavior implies a very large increase in the intensity of senior faculty usage, which would result in a considerable cost reduction per student.

(3) Intensify utilization of capital. Again in Table 5-2 the minimum level of classroom space is shown to be much less than the average level for institutions with the same enrollment and characteristics. This increased utilization of classroom space would also result in lower costs.

(4) Spread overhead by increasing the scale of operation. The effects of scale on the minimum level of senior faculty is shown in Table 5-14, and the effects of scale on minimum cost is shown in Table 5-20. Both relationships indicate that increasing scale from a very small level to a more average level results in a considerable decrease in costs and senior faculty, whereas increasing scale from an average level to a higher level does not result in a very significant decline in cost or senior faculty.

In addition to changes in how an institution produces, several results indicate the effect on costs of what an institution produces. Some of these results are listed below (the first two are from Bowen and Douglass, [1971]).

(1) Change the curricular mix. The relationship between the percent science degrees of total degrees granted and the minimum level of senior faculty is given in Table 5-14. Similarly, Table 5-20 shows the relationship between percent science degrees and the

minimum level of cost. Both of these relationships indicate the increase in costs resulting from a large proportion of science programs.

(2) Reduce noninstructional services. For the universities, Table 5-16 illustrates the relationship between the percent research revenues of total revenues and the minimum level of senior faculty and the relationship between the percent public service expenditures of total expenditures and the minimum level of senior faculty. The results indicate that a reduction in either of these percentages corresponds to a significant decline in the minimum level of senior faculty.

(3) Change the quality of the institution. The quality rating of the institution is shown in Tables 5-14 and 5-20 to be one of the strongest characteristic variables in affecting the minimum level of senior faculty and the minimum cost level.

(4) Change the number of programs offered. Program proliferation is expensive, as shown in Table 5-14. Institutions with more fields granting degrees (NFLD) have a much higher minimum level of senior faculty than institutions with few degree-granting fields or programs.

(5) Alter the enrollment mix. Marginal and average productivity relationships for changes in the level and mix of enrollment are given with respect to the minimum level of senior faculty in Tables 5-4 through 5-10 and with respect to minimum cost in Table 5-19. Due to the "jointness" between part-time and full-time undergraduates and all graduate students, the change in costs and/or senior faculty due to a change in the enrollment mix is a function of many variables. That is, the marginal cost of an additional graduate student is not a constant $XX regardless of the other characteristics and enrollments

of the institutions.

As evident from the above lists, the costs at higher education institutions can be changed in a variety of ways. It is important to note that the variation in costs resulting from what an institution produces is as large as the variation resulting from how an institution produces its outputs.

Institutional Diversity

Higher education as an industry probably exhibits a much greater diversity in behavior than most of the industries that are commonly studied within the framework of a production and cost analysis. This diversity does not mean that each higher education institution has a unique production process. Rather, it means that the production space for the higher education "industry" has many dimensions, and the institutions are spread widely throughout this space. The electricity-generating industry (M. Nerlove, [1963]), the railroad industry (G. H. Borts, [1958]), and the manufacturing industries (J. S. Bain, [1954]) are more likely to be composed of firms that produce relatively the same product with very few qualitative variations. Institutions of higher education, on the other hand, produce several different "products" with many qualitative variations. Although comparative analyses of other industries are not performed here, the framework of this study provides one means of illustrating the behavioral diversity of higher education institutions. This diversity has already been illustrated to some extent in Table 3-5. A different measure of diversity is obtained by solving the linear programming model for each institution i with cost in the objective function and the i^{th} institution as the right-hand side of the constraints. In addition, the i^{th} institution is deleted from the constraint matrix in order that it not enter into the final solution. Essentially, the LP has to construct the minimum cost institution with the same enrollment mix and set of characteristic variables as the i^{th} institution. If it is not possible to construct such an institution from the observations of all the other institutions in the sample, the constraints will be inconsistent and a solution infeasible. Whether the solution is feasible or infeasible is the information desired. If an institution's enrollment mix and characteristic variable set cannot be duplicated at any cost from the observed data, the solution will be infeasible and the institution's behavior is considerably different than that exhibited by any other or combination of other institutions.

Table 7-6 shows for each institutional category the total number of institutions in the sample, the number of institutions with infeasible solutions to the above LP problem, and the percent the latter are of the total number of institutions. As the results indicate, a substantial proportion of the institutions (the percentages range from 30% to 71%) cannot be "duplicated" from the observed variables of the other institutions. The wide diversity of behavior implied by these results suggests the hazard of assuming all institutions within so-called "homogeneous" categories are similar with respect to enrollment mix and characteristics. All of these factors should be taken into account in the analysis of cost, since, as the descriptive results of Chapter V indicate, all of these variables do have a strong effect on the production and cost relationships.

TABLE 7-6

Number of "Unique" Institutions by Institutional Type

Institutional Type	Total Institutions	"Unique" Institutions	Percent "Unique"
Public Universities	37	25	68
Private Universities	31	22	71
Public Comprehensive Colleges	105	44	42
Private Comprehensive Colleges	92	28	30
Public Limited Comprehensive Colleges	59	30	51
Private Highly Selective Liberal Arts Colleges	81	31	38
Private Less Selective Liberal Arts Colleges	269	*	

*Too expensive to compute; requires solving 269 relatively large linear programming problems.

VIII. SUMMARY AND CONCLUSIONS

As a means of summarizing the results from this empirical analysis of higher education institutions, several significant aspects of observed frontier production and cost behavior are listed and discussed below.

(1) Joint production processes: The descriptive results in Chapter V indicate that there are joint production relationships between enrollment by type and the institutional inputs. The transformation rates between part-time undergraduates, full-time undergraduates, and graduate students shown in Table 5-13 are not constant but depend on the relative proportions of each type of enrollment. The exception is the category of specialized enrollment which has a nearly constant transformation rate with full-time undergraduates for all institutional categories. These joint relationships are also illustrated by the marginal productivity results given in Tables 5-4 through 5-10 and by the average and marginal cost relationships given in Table 5-19. Joint relationships are also observed for the commitment to research and public service variables, as shown in Table 5-16. As the relative magnitude of one of the variables (part-time undergraduates, full-time undergraduates, graduate students, percent research revenues, percent public service expenditures) increases, the marginal cost of that variable also increases. For example, additional part-time students are more expensive for an institution with a larger proportion of part-time students.

(2) Variable productivities and costs: The average and marginal productivities and the average and marginal costs are complex functions

of the institution's input structure, enrollment mix, and characteristics. Two common production yardsticks in higher education, the student-faculty ratio and costs per student, are shown to vary considerably for frontier institutions with different enrollment mixes and sets of characteristic variables.

(3) Superfluous input levels: No substitution effects exist for institutional inputs above certain ratio levels. For the ratios of senior faculty versus junior faculty, general administrative personnel, and classroom space, additional amounts of the latter variables above certain levels do not correspond to reductions in the number of senior faculty on the production frontier. The critical levels vary across institutional categories and also depend on enrollment mixes and institutional characteristics to some degree.

(4) Frontier versus average: The input structures of the average-cost institutions versus the least-cost institutions indicate that the cost frontier is not a neutral transformation of the average cost relationships. Not only are the costs considerably lower on the frontier, but the input proportions also change significantly. It is interesting to note that the least-cost input structure does not correspond to the input structure with the lowest proportion of senior faculty. Therefore, simply increasing student-faculty ratios may not be the key to minimizing costs at institutions of higher education.

(5) Characteristic variables: All of the institutional characteristic variables included in the analysis have a significant effect on the production and cost behavior of these institutions. Quality, scale, program mix, number of fields, enrollment growth, and enrollment

retention strongly influence the average and marginal productivities and the average and marginal costs.

(6) Diversity in production and cost behavior: Due to several alternative input structures, many different enrollment mixes, and varying sets of institutional characteristics, over fifty percent of the institutions in each sample are needed to completely describe the observed production and cost frontiers. Since so many institutions are observed to be on the frontiers, it is most likely that some of these institutions are not truly efficient; they are only efficient relative to the other institutions. Therefore, the observed frontiers are most likely conservative estimates of the "true" efficient production and cost relationships.

(7) Distance from "average" to "frontier:" Even with conservative estimates of the frontier production and cost relationships, the average behavior is observed to be a considerable distance from the frontier. With all other variables held constant, the ratio of the average number of senior faculty to the frontier level of senior faculty for comparable institutions ranges from 1.20 to 1.76 across the institutional categories. Similarly, the ratio of the average cost per student to the frontier cost per student for institutions with comparable characteristics and enrollment mixes ranges from 1.42 to 2.20 across the categories of institutions.

(8) Behavior across types of institutions: The basic frontier production and cost relationships are very similar for each category of institutions analyzed, although the magnitudes of the relationships vary considerably. Even though the enrollment mixes and sets of characteristic variables are widely different between institutional

categories, an attempt is made to compare the public and private institutions of similar categories. Private institutions are always shown to have higher average costs per student, but these results show that for some enrollment mixes and sets of characteristic variables, the private institutions have lower costs per student than comparable public institutions. Comparisons across other categories are not feasible due to the diversity in behavior.

Implications of Results for Cost Analysis

The empirical results from this study summarized and discussed above have several implications for cost analysis in higher education. These implications are listed and discussed below.

(1) Inherent problems in comparing institutions: The results from this study show that many characteristics of higher education institutions strongly influence the institutions' cost and production behavior. Any comparison of costs across samples of institutions should be done cautiously to avoid labeling a high-cost institution inefficient if the high costs are the results of particular enrollment mixes and sets of characteristics. Comparing student-faculty ratios and costs-per-student across samples of institutions and ignoring other aspects of the institutions does not yield much useful information about the production and cost behavior of higher education institutions. As is evident from this analysis, many dimensions are needed in order to describe the behavior of colleges and universities, and all of these dimensions should be included in any analysis of cost and production relationships.

(2) Average versus frontier relationships: Given the distance

that is observed between the average relationships and the frontier relationships, a question arises: upon which relationships should policies be based? For example, should funds be allocated to institutions on the basis of average requirements or on the basis of frontier requirements? Also, since the frontier relationships appear to be non-neutral transformations of the average relationships, implications for the structure of inputs are different depending on which set of relationships is used. For efficiency and cost-minimizing reasons, the use of frontier relationships in policy formation is very appealing.

(3) Institutional missions and goals: The enrollment mix and the other characteristics of the institution were shown to strongly affect cost and production behavior. Since these variables are determined primarily by the mission and goals of the institution, it is important to consider *what* an institution is producing when one is analyzing costs. It appears that *what* an institution produces is as important as *how* the institution produces the output in determining the resulting cost and production behavior. Perhaps as much effort should go into studying why an institution has a particular enrollment mix and set of characteristics as goes into studying the input levels that determine the costs. One obvious way to lower higher education costs is to have "less expensive" enrollment mixes and institutional characteristics.

(4) Problems of analyzing subparts of the education process: The jointness observed between the various types of enrollment, research commitment, and public service commitment suggests that it is not possible to study any one of these higher education activities separately. Higher education institutions should be analyzed as joint production processes.

(5) Unit-cost studies: Since the observed average and marginal productivities and the observed average and marginal costs are complex functions of input structures, enrollment mixes, and institutional characteristics, the usefulness of constructing unit-costs becomes questionable. If unit-costs are to be used in institutional planning, then the changes in the institutional structure that are being proposed will result in changing the unit-costs. Unless unit-costs are derived as functions of all the other aspects of the institution, these cost estimates provide little aid in evaluating alternative plans. Also, since only one institution observed over time exhibits a very limited range of behavior, unit-cost estimates derived from one institutions's historical data will not apply to changes beyond the institution's limited range of experience. This diversity of behavior is one of the main advantages of cost and production relationships generated from cross-sectional data on a large number of institutions. With this source of data, the range of behavior is very large.

BIBLIOGRAPHY

Aigner, D. J. and S. F. Chu. "On Estimating the Industry Production Function." American Economic Review, September 1968, pp. 826-837.

American Council on Education. College and University Business Administration. Revised edition, 1968.

Astin, A. W. Predicting Academic Performance in College Selective Data for 2,300 American Colleges. American Council of Education, 1971.

________. "Undergraduate Achievement and Institutional Excellence." Science, August 16, 1968.

Bain, J. S. "Economies of Scale, Concentration, and Condition of Entry in Twenty Manufacturing Industries." American Economic Review, 1954, pp. 15-39.

Balderston, F. E. "Thinking About the Outputs of Higher Education." Ford Foundation Program for Research in University Administration, Paper P-5, University of California, Berkeley, 1970.

Baumol, W. "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis." American Economic Review, June 1967, pp. 415-426.

________ and W. G. Bowen. "On the Performing Arts: The Anatomy of their Economic Problems." American Economic Review, May 1965.

Bell, T. H. "The Means and Ends of Accountability." Proceedings of the Conference on Educational Accountability, March 1971.

Berelson, B. Graduate Education in the United States, 1960.

Berls, R. H. "An Explanation of the Determinants of Effectiveness in Higher Education." The Economics and Financing of Higher Education in the United States. A compendium of papers submitted to the Joint Economic Committee, 1969, pp. 207-260.

Blaug, M. Economics of Education 2. Penguin Books, 1969.

Boles, J. "The Measurement of Productive Efficiency: The Farrell Approach." Submitted for publication, University of California, Berkeley, 1972.

________. "The 1130 Farrell Efficiency System - Multiple Products, Multiple Factors." Giannini Foundation of Agricultural Economics, February 1971.

Bolton, R. E. "The Economics of Public Financing of Higher Education: An Overview." The Economics and Financing of Higher Education in the United States. A compendium of papers submitted to the Joint Economic Committee, 1969, pp. 11-106.

Borts, G. H. "The Estimation of Rail Cost Functions." Econometrica, January 1960, pp. 108-131.

Bowen, W. G. The Economics of the Major Private Universites, Carnegie Commission on Higher Education, McGraw-Hill, 1968.

Bowen, H. R. and C. K. Douglass. Efficiency in Liberal Education, Carnegie Commission on Higher Education, McGraw-Hill, 1971.

Bowles, S. "Towards an Educational Production Function." Education Income and Human Capital, edited by W. L. Hansen, 1970.

Brandl, J. E. "Public Service Outputs of Higher Education: An Exploratory Essay." Outputs of Higher Education, Western Interstate Commission on Higher Education, July 1970.

Breneman, D. W. "Internal Pricing Within the University - A Conference Report." Ford Foundation Program for Research in University Administration, Paper P-24, University of California, Berkeley, December 1971.

________. "The Ph.D. Production Function: The Case at Berkeley." Ford Foundation Program for Research in University Administration, Paper P-16, University of California, Berkeley, December 1970.

________. "The Stability of Faculty Input Coefficients in Linear Workload Models of the University of California." Ford Foundation Program for Research in University Administration, Paper 69-4, University of California, Berkeley, April 1969.

________ and G. B. Weathersby. "Definition and Measurement of the Activities and Outputs of Higher Education." Ford Foundation Program for Research in University Administration, Discussion Paper No. 10, University of California, Berkeley, August 1970.

Brown, D. B. "A Scheme for Measuring the Output of Higher Education." Outputs of Higher Education, Western Interstate Commission on Higher Education, July 1970.

Cain, G. G. and H. W. Watts. "Problems in Making Policy Inferences from the Coleman Report." American Sociological Review, April 1970, pp. 228-243.

California Coordinating Council for Higher Education. "Higher Cost Programs in California Public Higher Education." Report 71-3, March 1971.

California State Colleges. "A Unit Cost Study for San Fernando Valley State College, 1968-69." Office of the Chancellor, May 1971(a).

California State Colleges. "Preliminary Report on Institutional Cost Analysis." Committee on Educational Policy of the Division of Academic Planning, May 1971(b).

Carnegie Commission on Higher Education. Institutional Aid: Federal Support to Colleges and Universities. McGraw-Hill, 1972.

________. New Students, ~~New Places~~. McGraw Hill, 1971.

________. The More Effective Use of Resources: An Imperative for Higher Education. McGraw-Hill, 1972.

Carter, C. F. "The Efficiency of Universities." Higher Education, February 1972, pp. 77-90.

Cartter, A. An Assessment of Quality in Graduate Education. American Council on Education, 1966.

Casasco, J. A. Corporate Planning Models for University Management. ERIC Clearinghouse on Higher Education, Report 4, 1970.

Cavanaugh, A. D. A Preliminary Evaluation of Cost Studies in Higher Education. Office of Institutional Research, University of California, Berkeley, October 1969.

Cheit, E. F. The New Depression in Higher Education: A Study of Financial Conditions at 41 Colleges and Universities. Carnegie Commission on Higher Education, McGraw-Hill, 1971.

Columbia Research Associates. The Cost of College. October 1971.

Coombs, P. H. and J. Hallak. "Managing Education Costs." Oxford University Press, New York, 1972.

Dano, S. Industrial Production Models. 1966.

Diewert, W. E. "An Application of the Shephard Duality Theorem: A Generalized Leontif Production Function." Journal of Political Economy, May-June 1971, pp. 481-507.

________. "Functional Form in the Theory of Production and Consumer Demand." Unpublished dissertation, University of California, Berkeley, 1968.

Downey, L. W. "Alternative Policies and Strategies in the Financing of Post-Secondary Education." Prepared for The Council of Ministers of Education of Canada, May 1971.

Dressel, P. L. Institutional Research in the University: A Handbook. Jossey-Bass Inc., 1971.

Eads, G, M. Nerlove and W. Raduchel. "A Long-Run Cost Function for the Local Service Airline Industry: An Experiment in Non-Linear Estimation." Review of Economics and Statistics, August 1969, pp. 258-270.

Educational Technology. "Systems Techniques in Educational Planning and Management." February 1972, pp. 5-79.

Farrar, D. and R. Glauber. "Multicollinearity in Regression Analysis: The Problem Revisited." Review of Economics and Statistics, February 1967, pp. 92-107.

Farrell, M. J. "The Measurement of Productive Efficiency." Journal of the Royal Statistical Society - Series A, Part III, 1957, pp. 253-290.

________ and M. Fieldhouse. "Estimating Efficient Production Functions Under Increasing Returns to Scale." Journal of the Royal Statistical Society - Series A, Part II, 1962, pp. 252-267.

Feinstein, O. Higher Education in the United States - Economics, Personalism, Quality. 1971.

Feldstein, M. S. Economic Analysis for Health Service Efficiency. Markham Publishing Company, Chicago, 1968.

Fox, K. A. "Optimization Models for University Planning." Unpublished manuscript, March 1969.

Geoffrion, A. M., J. S. Dyer and A. Feinberg. "Academic Departmental Management: An Application of an Interactive Multi-Criterion Optimization Approach." Ford Foundation Program for Research in University Administration, Paper P-25, University of California, Berkeley, October 1971.

Gourman, J. The Gourman Report - Ratings of American Colleges. The Continuing Education Institute, Inc., 1967-68 edition.

Gulko, W. W. Program Classification Structure - Preliminary Edition for Review. Western Interstate Commission on Higher Education, June 1970.

________. "Unit Costs of Instruction: A Methodological Approach." Cost Finding Principles and Procedures, preliminary draft, Western Interstate Commission on Higher Education, 1971.

Hanoch, G. "Homotheticity in Joint Productions." Journal of Economics Theory, December 1970, pp. 423-426.

________ and M. Rothschild. "Testing the Assumptions of Production Theory - A Nonparametric Approach." Journal of Political Economy, March/April 1972, pp. 256-275.

Hansen, W. L. and B. A. Weisbrod. Benefits, Costs, and Finance of Public Higher Education. Markham Series in Public Policy Analysis, 1969.

Hansen, W. L., Kelley and B. A. Weisbrod. "Economic Efficiency and the Distribution of Benefits from College Instruction." American Economic Review, May 1970, pp. 364-369.

Henderson, J. and R. Quandt. Microeconomic Theory: A Mathematical Approach. McGraw-Hill, New York, 1958.

Hopkins, D. S. P. "On the Use of Large-Scale Simulation Models for University Planning." Unpublished manuscript, Stanford University, 1971.

Hough, R. "The Outputs of Undergraduate Education." Outputs of Higher Education, Western Interstate Commission on Higher Education, July 1970, pp. 93-104.

Jellema, W. W. The Red and the Black. 1971.

Jenny, H. and G. Wynn. The Golden Years. The College of Wooster, 1970.

________ and ________. The Turning Point. The College of Wooster, Ohio, 1972.

Johnson, C. B. and W. G. Katzenmeyer. Management Information Systems in Higher Education: The State of the Art. Duke University Press, 1969.

Joint Economic Committee. "The Economics and Financing of Higher Education in the United States." A compendium of papers submitted to the Congress of the United States, 1969.

Juster, F. "Microdata Requirements and Public Policy Designs." Annals of Economic and Social Measurement, January 1972.

Keller, J. F. "Higher Education Objectives: Measures of Performance and Effectiveness." Ford-Foundation Program for Research in University Administration, Paper P-7, University of California, Berkeley, May 1970.

Kendall, M. A Course in Multivariate Analysis. London, 1961.

Keniston, H. Graduate Study in the Humanities. The Educational Survey of the University of Pennsylvania, December 1957.

Kershaw, J. A. and A. M. Mood. "Resource Allocation in Higher Education." American Economic Review, May 1970, pp. 341-346.

Lau, L. J. and P. A. Yotopoulos. "A Test for Relative Efficiency and Application to Indian Agriculture." American Economic Review, March 1971, pp. 94-109.

Lave, J. R. and L. B. Lave. "Hospital Cost Functions." American Economic Review, June 1970, pp. 379-395.

Lawrence, B. "The Western Interstate Commission for Higher Education Information Systems Program." Management Information Systems in Higher Education, the State of the Art, edited by Johnson and Katzenmeyer, Chapter 6, 1969.

Lawrence, B. G. Weathersby and V. Patterson (eds.). Outputs of Higher Education: Their Identification, Measurement and Evaluation. Western Interstate Commission on Higher Education, July 1970.

Layard, P. R. G., J. D. Sargan, M. E. Ager and D. J. Jones. Qualified Manpower and Economic Performance. London School of Economics, Studies on Education, 1972.

Levin, H. "A Cost-Effectiveness Analysis of Teacher Selection." The Journal of Human Resources, Winter 1970, pp. 24-33.

________. "Concepts of Economic Efficiency and Educational Production." Preliminary manuscript, Stanford University, June 1971.

Martin, W. B. "The Problems of Size." Journal of Higher Education, March 1967, pp. 144-152.

Maynard, J. Some Microeconomics of Higher Education. 1971.

Millett, J. D. Planning, Programming and Budgeting for Ohio's Public Institutions of Higher Education. Ohio Board of Regents, May 1970.

Mundlak, Y. "Empirical Production Function Free of Management Bias." Journal of Farm Economics, February 1961, pp. 44-46.

________. "Specification and Estimation of Multiproduct Production Functions." Journal of Farm Economics, 1963, pp. 433-443.

Nerlove, M. Estimation and Identification of Cobb-Douglas Production Functions. Rand McNally and Co., Chicago, 1965.

________. "Further Evidence on the Estimation of Dynamic Economic Relations from a Time Series of Cross Sections." Econometrica, March 1971, pp. 359-382.

________. "Returns to Scale in Electricity Supply." Reprinted in A. Zellner, Readings in Economic Statistics of Econometrics, 1968.

Newhouse, J. P. "Toward a Theory of Non-profit Institutions: An Economic Model of a Hospital." American Economic Review, March 1970, pp. 64-74.

Newman, F. Report on Higher Education. March 1971.

Nordell, L. P. A Dynamic Input-Output Model of the California Educational System. Technical Report #25, Center for Research in Management Science, University of California, Berkeley, August 1967.

Office of the President. "Faculty Effort and Output Study." Report to the Committee on Educational Policy, University of California, Berkeley, January 9, 1970.

O'Neill, J. Resource Use in Higher Education: Trends in Output and Inputs, 1930 to 1967. Carnegie Commission on Higher Education, 1971.

Perl, L. J. "Graduation, Graduate School Attendance, and Investments in College Training." Ford Foundation Program for Research in University Administration, Paper P-21, University of California, Berkeley, July 1971.

Pfouts, R. "The Theory of Cost of Production in the Multiproduct Firm." Econometrica, October 1961, pp. 650-658.

Powell, J. and R. Lamson. Elements Related to the Determinants of Costs and Benefits of Graduate Education. 1972.

Radner, R. "Faculty-Student Ratios in United States Higher Education." Preliminary manuscript, June 1971.

Ramsey, J. and P. Zarembka. "Specification Error Tests and Alternative Functional Forms of the Aggregate Production Function." Journal of the American Statistical Association, September 1971, p. 471.

Reichard, D. J. Campus Size: A Selective Review. Southern Regional Education Board, 1971.

Ridker, R. and J. Henning. "The Determinants of Residential Property Values with Special Reference to Air Pollution." Review of Economics and Statistics, May 1967, pp. 246-257.

Schroeder, R. G. "A Survey of Operations Analysis in Higher Education." A paper prepared for the 41st National meeting of the Operations Research Society of America, April 1972.

Seitz, W. D. "Productive Efficiency in the Steam-Electric Generating Industry." Journal of Political Economy, July/August 1971, pp. 878-886.

________. "The Measurement of Efficiency Relative to a Frontier Production Function." American Journal of Agricultural Economics, November 1970, pp. 505-511.

Shubik, M. "On Different Methods for Allocating Resources." RAND Paper, P-4161, July 1969.

Sitorous, B. L. "Productive Efficiency and Redundant Factors of Production in Traditional Agriculture of Underdeveloped Countries." Proceedings, 39th Annual Meeting, Western Farm Economics Association, Los Angeles, California, 1966.

Smith, C. A. "Survey of the Empirical Evidence on Economies of Scale." Conference on Business Concentration and Price Policy, Princeton, 1955.

Smith, K. R., M. Miller and F. L. Golloday. "An Analysis of the Optimal Use of Inputs in the Production of Medical Services." The Journal of Human Resources, Spring 1972, pp. 208-225.

Smith, V. "More for Less: Higher Education's New Priority." Unpublished manuscript, July 1971.

Timmer, C. P. "Using a Probabilistic Frontier Production Function to Measure Technical Efficiency." Journal of Political Economy, July/August 1971, pp. 776-794.

Vinod, H. D. "Econometrics of Joint Production." Econometrica, April 1968, pp. 322-336.

Wagner, W. G. and G. B. Weathersby. "Optimality in College Planning: A Control Theoretic Approach." Ford Foundation Program for Research in University Administration, Paper P-22, University of California, Berkeley, December 1971.

Walters, A. A. "Production and Cost Functions: An Econometric Survey." Econometrica, January/April 1963.

Weathersby, G. B. "Educational Planning and Decision Making: The Use of Decision and Control Analysis." Ford Foundation Program for Research in University Administration, Paper P-6, University of California, Berkeley, May 1970.

________. "Tools and Techniques for Planning and Resource Allocation in Higher Education." Unpublished manuscript, January 1972.

________ and M. C. Weinstein. "A Structural Comparison of Analytical Models of University Planning." Ford Foundation Program for Research in University Administration, Paper P-12, University of California, Berkeley, August 1970.

Western Interstate Commission on Higher Education. Cost Finding Principles and Procedures. Preliminary Field Review Edition, TR-26, Boulder, Colorado.

________. "Faculty Activity Analysis: Overview of Major Issues." TR-24, December 1971.

________. "Objectives and Guidelines of the WICHE Management Information Systems Program." May 1969.

Wildavsky, A. "Rescuing Policy Analysis from PPBS." Public Administration Review, March/April 1969.

Wing, P. and M. Blumberg. "Operating Expenditures and Sponsored Research at U.S. Medical Schools: An Empirical Study of Cost Patterns." Journal of Human Resources, Winter 1971.

Winslow, F. D. "The Capital Cost of a University." Ford Foundation Program for Research in University Administration, Paper P-9, University of California, Berkeley, January 1971.

Witmer, D. R. "Cost Studies in Higher Education." Review of Educational Research, Vol. 42, #1, pp. 99-127.

Woodhall, M. "Cost-Benefit Analysis in Educational Planning." United Nation's Educational, Scientific, and Cultural Organization, 1970.

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